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A MADM Model with VIKOR Method for Decision Making Support Systems

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Abstract: **With respect to Multiple Attribute Group Decision Making (MAGDM) problems in which the attribute weights and attribute values take the form of the generalized interval-valued trapezoidal fuzzy numbers, a study is made on a new group decision making analysis called the VIKOR (VIšekriterijumsko KOmpromisno Rangiranje) method. An extended VIKOR method is presented to solve the MAGDM problems in which the attribute weights and values are given with the form of Generalized Interval Valued Trapezoidal Fuzzy Number (GIVTFN). An example is given to show the effectiveness of this method and decision making steps where three different distance functions are used to rank the alternatives.**

Keyword: **Decision Making, Decision Support Systems, MAGDM, VIKOR, Fuzzy Numbers, Trapezoidal Fuzzy Numbers, Generalized Trapezoidal Fuzzy Number, Generalized Interval Valued Trapezoidal Fuzzy Number.**

I. INTRODUCTION

In multiple attribute decision making (MADM) problem, a decision maker (DM) has to choose the best alternative that satisfies the evaluation criteria among a set of candidate solutions. It is generally hard to find an alternative that meets all the criteria simultaneously, so a good compromise solution is preferred. The VIKOR method was developed for multi-criteria optimization of complex systems. This method focuses on ranking and selecting from a set of alternatives in the presence of conflicting criteria. It introduces the multi-criteria ranking index based on the particular measure of ‘‘closeness” to the ‘‘ideal” solution. To deal with the uncertainty and vagueness from humans’ subjective perception and experience in decision process, this paper presents an evaluation model based on deterministic data, fuzzy numbers, interval numbers and linguistic terms. More on imprecise data or fuzzy data are discussed in [2-4,6,9,11,15,25,27,31,34,35].Multi-criteria optimization is the process of determining the best feasible solution according to the established criteria (representing different effects). Practical problems are often characterized by several non-commensurable and conflicting criteria and there may be no solution satisfying all criteria simultaneously. Thus, the solution is a set of non-inferior solutions, or a compromise solution according to the decision maker’s preferences. The compromise solution was established by Zeleny [36] for a problem with conflicting criteria and it can help the decision makers to reach a final solution. In classical MADM methods, the ratings and the weights of the criteria are known precisely, whereas in the real world, in an imprecise and uncertain environment, it is an unrealistic assumption that the knowledge and representation of a decision maker or expert are so precise. For example, human judgment including preferences is often vague and decision maker (DM) cannot estimate his preference with exact numerical values. In these situations, determining the exact value of the attributes is difficult or impossible. So, to describe and treat imprecise and uncertain elements present in a decision problem, fuzzy approaches and linguistic terms are frequently used. In the works of linguistic terms decision making, linguistic terms are assumed to be with known by fuzzy linguistic membership function. However, in reality to a decision maker it is not always easy to specify the membership function in an inexact environment. At least in some of the cases, the use of interval numbers may serve the purpose better. An interval number can be thought as an extension of the concept of a real number, however, in decision problems its use is not much attended as it merits. Recently, some authors have extended TOPSIS and VIKOR method to solve decision making problems with interval data. According to a comparative analysis of VIKOR and TOPSIS written by Opricovic & Tzeng [20], VIKOR and TOPSIS methods use different aggregation functions and different normalization methods. Many other authors also have worked considerably on VIKOR and TOPSIS methods [16,17,20,21,26,30,32,33]. TOPSIS method is based on the principle that the optimal point should have the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS). Therefore, this method is suitable for cautious (risk avoider) decision maker(s), because the decision maker(s) might like to have a decision which not only makes as much profit as possible, but also avoids as much risk as possible. Besides, computing the optimal point in the VIKOR is based on the particular measure of ‘‘closeness” to the PIS. Therefore, it is suitable for those situations in which the decision maker wants to have maximum profit and the risk of the decisions is less important for him/her. Therefore, in this paper, VIKOR method was extended to develop a methodology for solving MADM problems. What we basically suggest in this study is to extend the VIKOR method with four main types of information (deterministic data, fuzzy numbers, interval numbers and linguistic terms) in decision-making matrix for solving multiple attribute decision making problems. To validate the application of the model and to examine its effectiveness, the proposed extension methodology together with three distance functions is compared.

II. DECISION SUPPORT SYSTEMS (DSS) AND TECHNIQUES

Decision making, by its nature, is a cognitive process, involving different cognitive tasks, such as collecting information, evaluating situation, generating and selecting alternatives, and implementing solutions [5,12,15,18,19,28,29,34,36]. Decision making is never error-proof, as decision makers are prone to their cognitive biases. Therefore, decision support systems (DSS) are often used by decision makers in order to minimize their cognitive errors and maximize the performance of actions. A properly-designed DSS can play an important role in compiling useful information from raw data, documents, personal knowledge, and business models to solve problems [26]. It allows decision makers to perform large numbers of computations very quickly. Therefore advanced models can be supported by DSS to solve complex decision problems. As many business decision problems involve large data sets stored in different databases, data warehouses, and even possibly at websites outside an organization, DSS can retrieve process and utilize data efficiently to assist decision making. A DSS is intended to support, rather than replace, decision maker's role in solving problems. Decision makers' capabilities are extended through using DSS, particularly in ill-structured decision situations. In this case, a satisfied solution, instead of the optimal one, may be the goal of decision making. Solving ill-structured problems often relies on repeated interactions between the decision maker and the DSS. Decision support systems are built upon various decision support techniques, including models, methods, algorithms and tools. A cognition-based taxonomy for decision support techniques, including six basic classes as follows: Process models, Choice models, Information control techniques, Analysis and reasoning techniques, Representation aids and Human judgment amplifying/refining techniques. The Multi-criteria decision making and Multi-attribute decision making comes under the category of Choice models. In the Literature ([1,7,8,10,13,14,22,23,24,26]) various authors have worked on DSS models involving decision making problems.

A. Application of VIKOR as a Decision Support Technique

Multiple Attribute decision support systems are provided to assist decision makers with an explicit and comprehensive tool and techniques in order to evaluate alternatives in terms of different factors and importance of their weights. Some of the common Multi-Attribute Decision-Making (MADM) techniques are [7, 26]:

- Simple Additive Weighted (SAW)
- Weighted Product Method (WPM)
- Cooperative Game Theory (CGT)
- Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)
- Elimination et Choice Translating Reality with complementary analysis(ELECTRE)
- Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE)
- Analytical Hierarchy Process (AHP)
- VIšekriterijumsko KOmpromisno Rangiranje (VIKOR)

The merit of the VIKOR method suggested is that it can deal with both quantitative and qualitative assessment in the process evaluation with little computation load. It bases upon the concept that the chosen alternative should have the compromise solution. In the process of VIKOR, the performance ratings and the weights of the criteria are given as crisp values. In fuzzy VIKOR, attribute values are represented by fuzzy numbers.

B. The VIKOR Method

Decision-making problem is the process of finding the best option from all of the feasible alternatives. In almost all such problems, the multiplicity of criteria for judging the alternatives is pervasive. For many such problems, the DM wants to solve a multiple attribute decision making (MADM) problem [12]. A MADM problem can be concisely expressed in matrix format as:

$$\begin{array}{cc} & C_n \\ A_m & \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \cdots & \cdots & & \cdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \end{array}$$

where $A_1, A_2, \ldots, A_m$ are possible alternatives among which decision makers have to choose, $C_1, C_2, \ldots, C_n$ are criteria with which alternative performance are measured, x_{ij} is the rating of alternative A_i with respect to criterion C_j. The foundation for compromise solution was established in [36] and later advocated in [18-21]. The compromise solution is a feasible solution that is the closest to the ideal solution, and a compromise means an agreement established by mutual concession. The compromise solution method, also known as the VIKOR method was introduced as one applicable technique to implement within MADM. The multiple attribute merit for compromise ranking was developed from the L*p*-metric used in the compromise programming method [36]. The main procedure of the VIKOR method is described below:

Step 1: The first step is to determine the objective, and to identify the pertinent evaluation attributes. Also determine the best, i.e., f_j^+ and the worst, f_j^-, values of all attributes.

Step 2: Calculate the values of S_i and R_i :

$$S_i = \sum_{j=1}^{M} w_j \left([f_j^+ - f_{ij}] / [f_j^+ - f_j^-] \right), \quad R_i = \underset{j}{Max} \left\{ w_j \left([f_j^+ - f_{ij}] / [f_j^+ - f_j^-] \right) \right\}, \; j = 1, 2, \ldots, M.$$

Step 3: Calculate the values of Q_i :

$$Q_i = v \frac{(S_i - S^*)}{(S^- - S^*)} + (1 - v) \frac{(R_i - R^*)}{(R^- - R^*)}, \qquad where \;\; S^* = \min_i S_i \;\; , \;\; S^- = \max_i S_i$$

where S^* is the maximum value of S_i, and S^- the minimum value of S_i; R^* is the maximum value of R_i, and R^- is the minimum value of R_i . v is introduced as weight of the strategy of 'the majority of attributes'. Usually, the value of v is taken as 0.5. However, v can take any value from 0 to 1.

Step 4: Arrange the alternatives in the descending order, according to the values of Q_i. Similarly, arrange the alternatives according to the values of S_i and R_i separately. Thus, three ranking lists can be obtained. The compromise ranking list for a given v is obtained by ranking with Q_i measures. The best alternative,

ranked by Q_i, is the one with the minimum value of Q_i.

Step 5: For given attribute weights, propose a compromise solution, alternative A1, which is the best ranked by the measure Q, if the following two conditions are satisfied:

Condition 1: 'Acceptable advantage' $Q(A_2) - Q(A_1) \geq (1/(N-1))$.

A_2 is the second-best alternative in the ranking by Q.

Condition 2: 'Acceptable stability in decision making'. Alternative A_1 must also be the best ranked by S and/or R. This compromise solution is stable within a decision-making process, which could be: 'voting by majority rule'

(when $v > 0.5$ is needed) or 'by consensus' (when $v = 0.5$) or 'with veto' (when $v < 0.5$).

If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

– Alternatives A_1 and A_2 if only condition 2 is not satisfied.

– Alternatives $A_1, A_2, \ldots, A_m$ if condition 1 is not satisfied; A_m is determined by the relation $Q(A_m) - Q(A_l) < (1/(N - 1))$ for maximum M (the positions of these alternatives are ''in closeness'').

VIKOR is a helpful tool in MADM, particularly in a situation where the decision maker is not able, or does not know how to express preference at the beginning of system design. The obtained compromise solution could be accepted by the decision makers because it provides a maximum 'group utility' (represented by S^-) of the 'majority' and a minimum of individual regret (represented by R^-) of the 'opponent'.

III. ILLUSTRATIVE EXAMPLE

Supposes that a telecommunication company intends to choose a manager for R&D department from four volunteers named a_1, a_2, a_3 and a_4. The decision making committee assesses the four concerned volunteers based on attributes which follows :

1. proficiency in identifying research areas(c_1)
2. proficiency in administration(c_2),
3. personality(c_3)
4. past experience(c_4),

Table.1 Linguistic terms with corresponding generalized interval valued trapezoidal fuzzy numbers

Linguistic items (the attributes values)	Linguistic items (weights)	Generalized interval valued trapezoidal fuzzy numbers
Absolutely- poor(AP)	Absolutely-low(AL)	[(0.00,0.00,0.00,0.00;0.8),(0.00,0.00,0.00,0.00;1.0)]
Very-poor(VP)	Very-low(VL)	[(0.00,0.00,0.02,0.07;0.8),(0.00,0.00,0.02,0.07;1.0)]
Poor(P)	Low(L)	[(0.04,0.10,0.18,0.23;0.8),(0.04,0.10,0.18,0.23;1.0)]
Medium-poor(MP)	Medium-low (ML)	[(0.17,0.22,0.36,0.42;0.8),(0.17,0.22,0.36,0.42;1.0)]
Medium(F)	Medium(M)	[(0.32,0.41,0.58,0.65;0.8),(0.32,0.41,0.58,0.65;1.0)]
Medium-good(MG)	Medium-high(MH)	[(0.58,0.63,0.80,0.86;0.8),(0.58,0.63,0.80,0.86;1.0)]
Good(G)	High (H)	[(0.72,0.78,0.92,0.97;0.8),(0.72,0.78,0.92,0.97;1.0)]
Very-good(VG)	Very-High(VH)	[(0.93,0.98,1.00,1.00;0.8),(0.93,0.98,1.00,1.00;1.0)]
Absolutely good(AG)	Absolutely- High(AH)	[(1.00,1.00,1.00,1.00;0.8),(1.00,1.00,1.00,1.00;1.0)]

Table.2The attributes weights given by three Decision makers,

	C_1	C_2	C_3	C_4	C_5
DM_1	VH	H	H	VH	M
DM_2	VH	H	MH	H	MH
DM_3	VH	MH	MH	VH	M

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Table.3 The evaluation information of four volunteers given by,

	C_1	C_2	C_3	C_4	C_5
a_1	VG	VG	VG	VG	VG
a_2	G	VG	VG	VG	MG
a_3	VG	MG	G	G	G
a_4	G	F	F	G	MG

Table.4 The evaluation information of four volunteers given by,

	C_1	C_2	C_3	C_4	C_5
a_1	MG	F	G	VG	VG
a_2	MG	MG	G	MG	G
a_3	VG	VG	VG	VG	MG
a_4	MG	VG	MG	VG	F

Table.5 The evaluation information of four volunteers given by,

	C_1	C_2	C_3	C_4	C_5
a_1	MG	F	G	VG	VG
a_2	MG	MG	G	MG	G
a_3	VG	VG	VG	VG	MG
a_4	MG	MG	MG	VG	F

$$\left[\omega_{ij}\right]_{3\times5} = \left[\begin{matrix} \left[(0.93,0.98,1.00,1.00;0.8)(0.93,0.98,1.00,1.00,1.00)\right] \\ \left[(0.93,0.98,1.00,1.00;0.8)(0.93,0.98,1.00,1.00,1.00)\right] \\ \left[(0.93,0.98,1.00,1.00;0.8)(0.93,0.98,1.00,1.00,1.00)\right] \end{matrix}\right.$$

$$\begin{matrix} \left[(0.72,0.78,0.92,0.97;0.8)(0.72,0.78,0.92,0.97;1.00)\right] \\ \left[(0.72,0.78,0.92,0.97;0.8)(0.72,0.78,0.92,0.97;1.00)\right] \\ \left[(0.58,0.63,0.80,0.86;0.8)(0.58,0.63,0.80,0.86;1.00)\right] \end{matrix}$$

$$\begin{matrix} \left[(0.72,0.78,0.92,0.97;0.8)(0.72,0.78,0.92,0.97;1.00)\right] \\ \left[(0.58,0.63,0.80,0.86;0.8)(0.58,0.63,0.80,0.86;1.00)\right] \\ \left[(0.58,0.63,0.80,0.86;0.8)(0.58,0.63,0.80,0.86;1.00)\right] \end{matrix}$$

$$\begin{matrix} \left[(0.93,0.98,1.00,1.00\ ;0.8)(0.93,0.98,1.00,1.00\ ;1.00)\right] \\ \left[(0.72,0.78,0.92,0.97;0.8)(0.72,0.78,0.92,0.97;1.00)\right] \\ \left[(0.93,0.98,1.00,1.00\ ;0.8)(0.93,0.98,1.00,1.00\ ;1.00)\right] \end{matrix}$$

$$\left.\begin{matrix} \left[(0.32,0.41,0.58,0.65;0.8)(0.32,0.41,0.58,0.65;1.00)\right] \\ \left[(0.58,0.63,0.80,0.86;0.8)(0.58,0.63,0.80,0.86;1.00)\right] \\ \left[(0.32,0.41,0.58,0.65;0.8)(0.32,0.41,0.58,0.65;1.00)\right] \end{matrix}\right]$$

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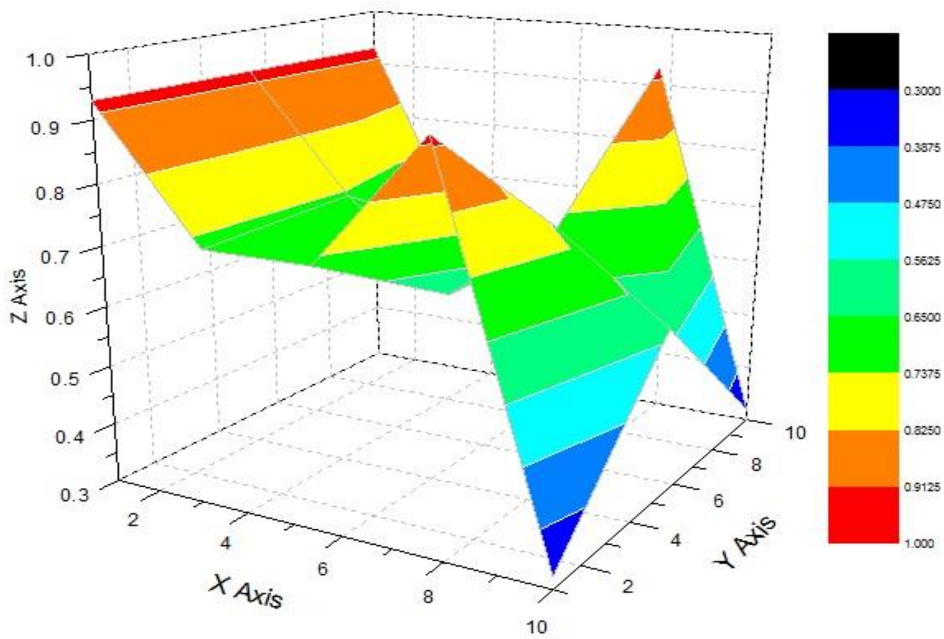

Fig.1 Three dimensional representation of the weight matrix

$$\left[\tilde{\tilde{a}}_{ij1}\right]_{4\times 5}=\begin{bmatrix}\left[(0.93,0.98,1.00,1.00;\ 0.8)(0.93,0.98,1.00,1.00;\ 1.00)\right]\\ \left[(0.72,0.78,0.92,0.97;0.8)(0.72,0.78,0.92,0.97;1.00)\right]\\ \left[(0.93,0.98,1.00,1.00;\ 0.8)(0.93,0.98,1.00,1.00;\ 1.00)\right]\\ \left[(0.72,0.78,0.92,0.97;0.8)(0.72,0.78,0.92,0.97;1.00)\right]\end{matrix}$$

$$\begin{matrix}\left[(0.93,0.98,1.00,1.00;\ 0.8)(0.93,0.98,1.00,1.00;\ 1.00)\right]\\ \left[(0.93,0.98,1.00,1.00;\ 0.8)(0.93,0.98,1.00,1.00;\ 1.00)\right]\\ \left[(0.58,0.63,0.80,0.86;\ 0.8)(0.58,0.63,0.80.0.86;\ 1.00)\right]\\ \left[(0.32,0.41,0.58,0.65;\ 0.8)(0.32,0.41,0.58,0.65;\ 1.00)\right]\end{matrix}$$

$$\begin{matrix}\left[(0.93,0.98,1.00,1.00;\ 0.8)(0.93,0.98,1.00,1.00;\ 1.00)\right]\\ \left[(0.93,0.98,1.00,1.00;\ 0.8)(0.93,0.98,1.00,1.00;\ 1.00)\right]\\ \left[(0.72,0.78,0.92,0.97;0.8)(0.72,0.78,0.92,0.97;1.00)\right]\\ \left[(0.32,0.41,0.58,0.65;0.8)(0.32,0.41,0.58,0.65;\ 1.00)\right]\end{matrix}$$

$$\begin{matrix}\left[(0.93,0.98,1.00,1.00;\ 0.8)(0.93,0.98,1.00,1.00;\ 1.00)\right]\\ \left[(0.93,0.98,1.00,1.00;\ 0.8)(0.93,0.98,1.00,1.00;\ 1.00)\right]\\ \left[(0.72,0.78,0.92,0.97;0.8)(0.72,0.78,0.92,0.97;1.00)\right]\\ \left[(0.72,0.78,0.92,0.97;0.8)(0.72,0.78,0.92,0.97;1.00)\right]\end{matrix}$$

$$\begin{bmatrix}\left[(0.93,0.98,1.00,1.00;\ 0.8)(0.93,0.98,1.00,1.00;\ 1.00)\right]\\ \left[(0.58,0.63,0.80,0.86;\ 0.8)(0.58,0.63,0.80.0.86;\ 1.00)\right]\\ \left[(0.72,0.78,0.92,0.97;0.8)(0.72,0.78,0.92,0.97;1.00)\right]\\ \left[(0.58,0.63,0.80,0.86;\ 0.8)(0.58,0.63,0.80.0.86;\ 1.00)\right]\end{bmatrix}$$

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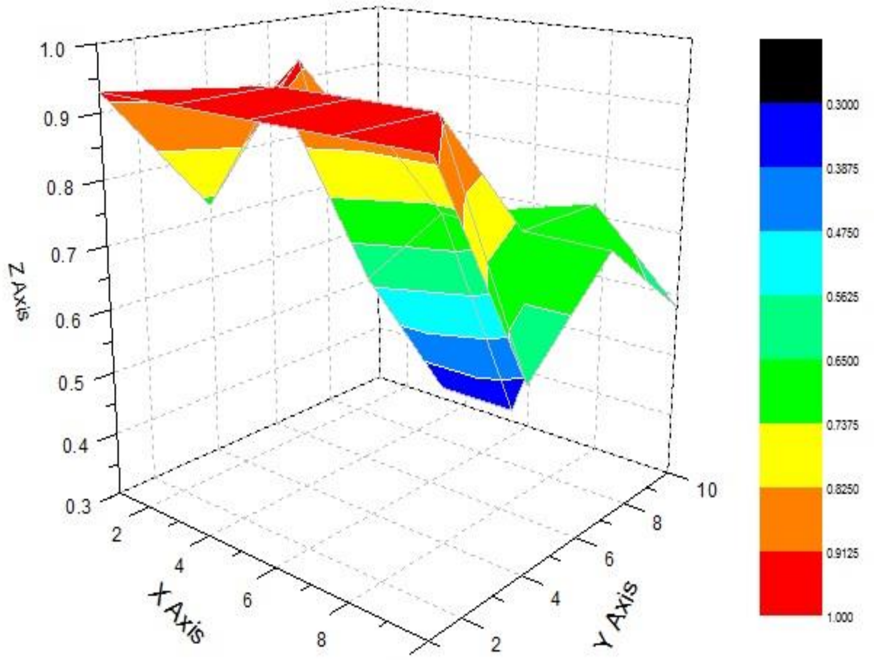

Fig.2 Three dimensional representation of the decision matrix 1

$$\left[\tilde{\tilde{a}}_{ij2}\right]_{4\times 5}=\left[\begin{array}{ccccc}
\left[(0.72,0.78,0.92,0.97;0.8)(0.72,0.78,0.92,0.97;1.00)\right] & \left[(0.58,0.63,0.80,0.86;\ 0.8)(0.58,0.63,0.80.0.86;\ 1.00)\right] & \left[(0.72,0.78,0.92,0.97;0.8)(0.72,0.78,0.92,0.97;1.00)\right] & \left[(0.72,0.78,0.92,0.97;0.8)(0.72,0.78,0.92,0.97;1.00)\right] & \left[(0.93,0.98,1.00,1.00;\ 0.8)(0.93,0.98,1.00,1.00;\ 1.00)\right] \\
\left[(0.72,0.78,0.92,0.97;0.8)(0.72,0.78,0.92,0.97;1.00)\right] & \left[(0.93,0.98,1.00,1.00;\ 0.8)(0.93,0.98,1.00,1.00;\ 1.00)\right] & \left[(0.93,0.98,1.00,1.00;\ 0.8)(0.93,0.98,1.00,1.00;\ 1.00)\right] & \left[(0.93,0.98,1.00,1.00;\ 0.8)(0.93,0.98,1.00,1.00;\ 1.00)\right] & \left[(0.58,0.63,0.80,0.86;\ 0.8)(0.58,0.63,0.80.0.86;\ 1.00)\right] \\
\left[(0.72,0.78,0.92,0.97;0.8)(0.72,0.78,0.92,0.97;1.00)\right] & \left[(0.72,0.78,0.92,0.97;0.8)(0.72,0.78,0.92,0.97;1.00)\right] & \left[(0.58,0.63,0.80,0.86;\ 0.8)(0.58,0.63,0.80.0.86;\ 1.00)\right] & \left[(0.93,0.98,1.00,1.00;\ 0.8)(0.93,0.98,1.00,1.00;\ 1.00)\right] & \left[(0.72,0.78,0.92,0.97;0.8)(0.72,0.78,0.92,0.97;1.00)\right] \\
\left[(0.93,0.98,1.00,1.00;\ 0.8)(0.93,0.98,1.00,1.00;\ 1.00)\right] & \left[(0.32,0.41,0.58,0.65;0.8)(0.32,0.41,0.58,0.65;\ 1.00)\right] & \left[(0.58,0.63,0.80,0.86;\ 0.8)(0.58,0.63,0.80.0.86;\ 1.00)\right] & \left[(0.32,0.41,0.58,0.65;\ 0.8)(0.32,0.41,0.58,0.65;1.00)\right] & \left[(0.72,0.78,0.92,0.97;0.8)(0.72,0.78,0.92,0.97;1.00)\right]
\end{array}\right]$$

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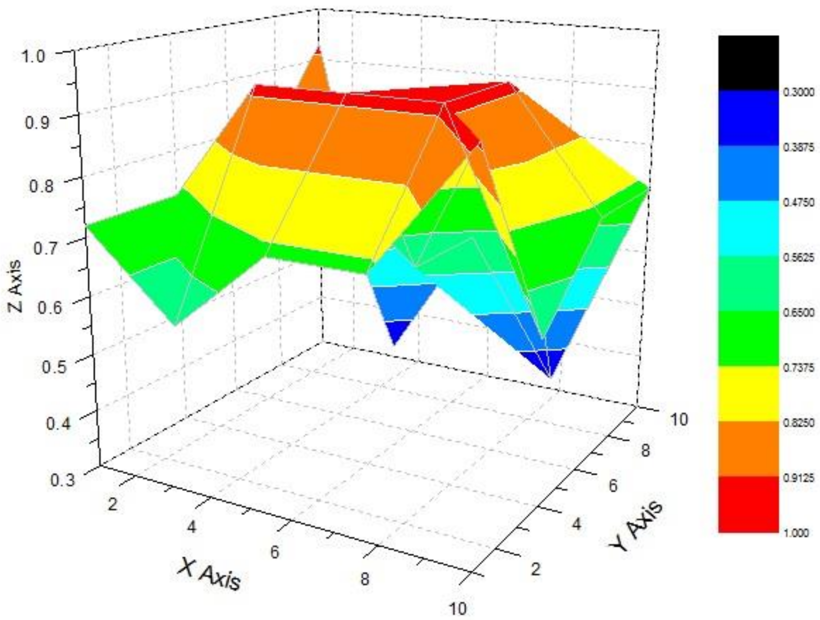

Fig.3 Three dimensional representation of the decision matrix 2

$$\left[\tilde{\tilde{a}}_{ij3}\right]_{4\times5} = \begin{bmatrix} [(0.58,0.63,0.80,0.86;0.8)(0.58,0.63,0.80,0.65;\ 1.00)] & [(0.32,0.41,0.58,0.65;\ 0.8)(0.32,0.41,0.58,0.65;1.00)] & [(0.72,0.78,0.92,0.97;0.8)(0.72,0.78,0.92,0.97;1.00)] & [(0.93,0.98,1.00,1.00;\ 0.8)(0.93,0.98,1.00,1.00;\ 1.00)] & [(0.93,0.98,1.00,1.00;\ 0.8)(0.93,0.98,1.00,1.00;\ 1.00)] \\ [(0.58,0.63,0.80,0.86;0.8)(0.58,0.63,0.80,0.65;\ 1.00)] & [(0.58,0.63,0.80,0.86;\ 0.8)(0.58,0.63,0.80,0.65;\ 1.00)] & [(0.72,0.78,0.92,0.97;0.8)(0.72,0.78,0.92,0.97;1.00)] & [(0.58,0.63,0.80,0.86;\ 0.8)(0.58,0.63,0.80,0.65;\ 1.00)] & [(0.72,0.78,0.92,0.97;0.8)(0.72,0.78,0.92,0.97;1.00)] \\ [(0.93,0.98,1.00,1.00;\ 0.8)(0.93,0.98,1.00,1.00;\ 1.00)] & [(0.93,0.98,1.00,1.00;\ 0.8)(0.93,0.98,1.00,1.00;\ 1.00)] & [(0.93,0.98,1.00,1.00;\ 0.8)(0.93,0.98,1.00,1.00;\ 1.00)] & [(0.93,0.98,1.00,1.00;\ 0.8)(0.93,0.98,1.00,1.00;\ 1.00)] & [(0.58,0.63,0.80,0.86;0.8)(0.58,0.63,0.80,0.65;1.00)] \\ [(0.93,0.98,1.00,1.00;\ 0.8)(0.93,0.98,1.00,1.00;\ 1.00)] & [(0.93,0.98,1.00,1.00;\ 0.8)(0.93,0.98,1.00,1.00;\ 1.00)] & [(0.58,0.63,0.80,0.86;\ 0.8)(0.58,0.63,0.80,0.65;1.00)] & [(0.93,0.98,1.00,1.00;\ 0.8)(0.93,0.98,1.00,1.00;\ 1.00)] & [(0.32,0.41,0.58,0.65;0.8)(0.32,0.41,0.58,0.65;1.00)] \end{bmatrix}$$

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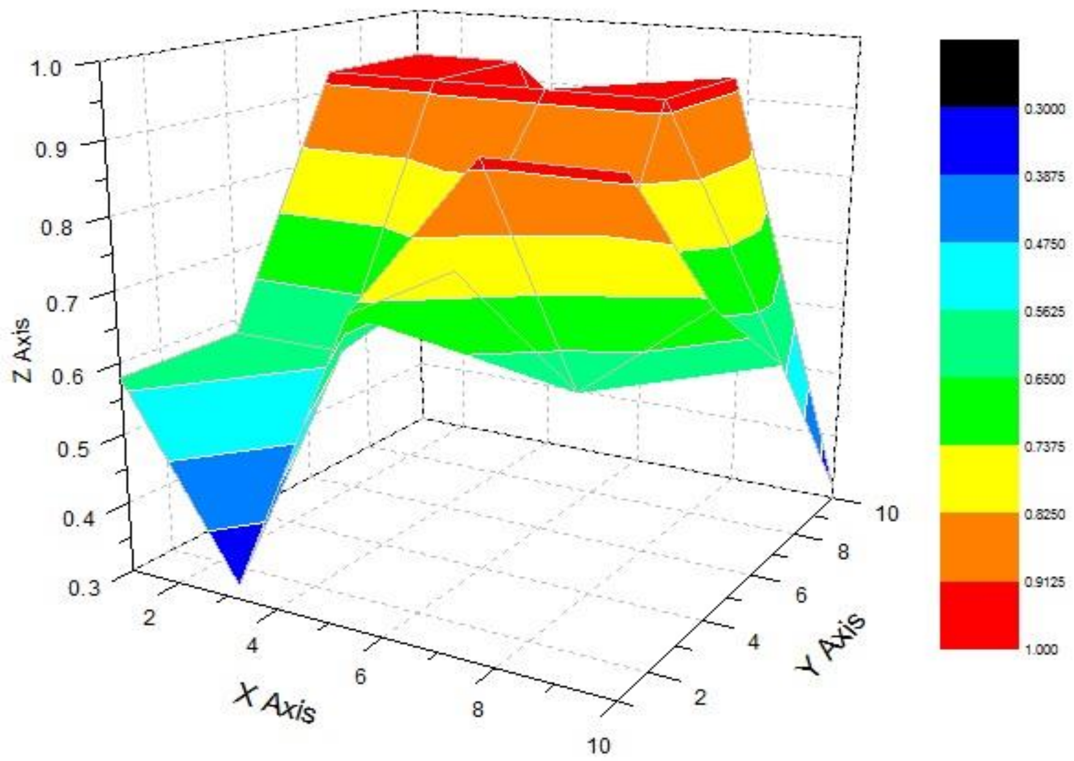

Fig.4 Three dimensional representation of the decision matrix 3

Step-2:

Combine the individual preferences in order to obtain a collective preference value for each alternative:

$$\tilde{\tilde{x}}_{ij} = [(x_{ij1}{}^L, x_{ij2}{}^L, x_{ij3}{}^L, x_{ij4}{}^L; w_{ij}{}^L), (x_{ij1}{}^U, x_{ij2}{}^U, x_{ij3}{}^U, x_{ij4}{}^U; w_{ij}{}^U)]$$

$$= \sum_{k=1}^{q} (\lambda_k \tilde{\tilde{x}}_{ij_k}), q = 3, \sum \lambda_k = 1, \lambda = 1, 2, 3, \ldots$$

$$= \sum_{k=1}^{3} [\lambda_k (x_{ijk1}{}^L, x_{ijk2}{}^L, x_{ijk3}{}^L, x_{ijk4}{}^L; w_{ij}{}^L), (x_{ijk1}{}^U, x_{ijk2}{}^U, x_{ijk3}{}^U, x_{ijk4}{}^U; w_{ij}{}^U)]$$

$$= \lambda_1 [(x_{ijk1}{}^L, x_{ijk2}{}^L, x_{ijk3}{}^L, x_{ijk4}{}^L; w_{ij}{}^L), (x_{ijk1}{}^U, x_{ijk2}{}^U, x_{ijk3}{}^U, x_{ijk4}{}^U; w_{ij}{}^U)]$$

$$+\lambda_2 [(x_{ijk1}{}^L, x_{ijk2}{}^L, x_{ijk3}{}^L, x_{ijk4}{}^L; w_{ij}{}^L), (x_{ijk1}{}^U, x_{ijk2}{}^U, x_{ijk3}{}^U, x_{ijk4}{}^U; w_{ij}{}^U)]$$

$$+\lambda_3 [(x_{ijk1}{}^L, x_{ijk2}{}^L, x_{ijk3}{}^L, x_{ijk4}{}^L; w_{ij}{}^L), (x_{ijk1}{}^U, x_{ijk2}{}^U, x_{ijk3}{}^U, x_{ijk4}{}^U; w_{ij}{}^U)]$$

We get,

$$\left[\tilde{\tilde{x}}_{ij}\right]_{4\times 5} = \begin{bmatrix} \left[(0.741, 0.759, 0.908, 0.946; 0.800), (0.741, 0.795, 0.908, 0.946; 1.000)\right] \\ \left[(0.678, 0.735, 0.884, 0.937; 0.800), (0.678, 0.735, 0.884, 0.937; 1.000)\right] \\ \left[(0.846, 0.900, 0.968, 0.988; 0.800), (0.846, 0.900, 0.968, 0.988; 1.000)\right] \\ \left[(0.762, 0.815, 0.916, 0.941; 0.800), (0.762, 0.815, 0.916, 0.941; 1.000)\right] \end{bmatrix}$$

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$$\begin{matrix} \left[(0.607,0.669,0.794,0.839,0.800),(0.607,0.669,0.794,0.839;1.000)\right] \\ \left[(0.825,0.875,0.940,0.958;0.800),(0.825,0.875,0.940,0.958;1.000)\right] \\ \left[(0.741,0.795,0.907,0.946;0.800),(0.741,0.795,0.907,0.946;1.000)\right] \\ \left[(0.503,0.581,0.706,0.755;0.800),(0.503,0.581,0.706,0.755;1.000)\right] \end{matrix}$$

$$\begin{matrix} \left[(0.783,0.847,0.947,0.979;0.800),(0.783,0.847,0.947,0.979;1.000)\right] \\ \left[(0.867,0.920,0.976,0.991;0.800),(0.867,0.920,0.976,0.991;1.000)\right] \\ \left[(0.727,0.780,0.896,0.935;0.800),(0.727,0.780,0.896,0.935;1.000)\right] \\ \left[(0.502,0.564,0.734,0.790;0.800),(0.502,0.564,0.734,0.790;1.000)\right] \end{matrix}$$

$$\begin{matrix} \left[(0.846,0.900,0.968,0.988;0.800),(0.846,0.900,0.968,0.988;1.000)\right] \\ \left[(0.825,0.875,0.940,0.958;0.800),(0.825,0.875,0.940,0.958;1.000)\right] \\ \left[(0.867,0.920,0.976,0.994;0.800),(0.867,0.920,0.976,0.994;1.000)\right] \\ \left[(0.623,0.692,0.808,0.854;0.800),(0.623,0.692,0.808,0.854;1.000)\right] \end{matrix}$$

$$\begin{matrix} \left[(0.622,0.675,0.836,0.893;0.800),(0.622,0.675,0.836,0.893;1.000)\right] \\ \left[(0.622,0.675,0.836,0.893;0.800),(0.622,0.675,0.836,0.893;1.000)\right] \\ \left[(0.678,0.735,0.884,0.937;0.800),(0.678,0.735,0.884,0.937;1.000)\right] \\ \left[(0.558,0.624,0.782,0.841;0.800),(0.558,0.624,0.782,0.841;1.000)\right] \end{matrix}\Bigg]$$

Calculate $\left[\tilde{\tilde{\omega}}_j\right]_{1\times 5}$:

$$\left[\tilde{\tilde{\omega}}_j\right]_{1\times 5} = \Big(\left[(0.930,0.980,1.000,1.000;0.800),(0.930,0.980,1.000,1.000;1.000)\right]$$
$$\left[(0.678,0.735,0.884,0.937;0.800),(0.678,0.735,0.884,0.937;1.000)\right]$$
$$\left[(0.622,0.675,0.836,0.893;0.800),(0.622,0.675,0.836,0.893;1.000)\right]$$
$$\left[(0.846,0.900,0.968,0.988;0.800),(0.846,0.900,0.968,0.988;1.000)\right]$$
$$\left[(0.424,0.498,0.668,0.734;0.800),(0.424,0.498,0.668,0.734;1.000)\right]\Big)$$

Step-3: Calculate the weighted decision making matrix :

$$\left[\tilde{\tilde{v}}_{ij}\right] = [(v_{ij1}{}^{L}, v_{ij2}{}^{L}, v_{ij3}{}^{L}, v_{ij4}{}^{L}; w_{ij}{}^{L}), (v_{ij1}{}^{U}, v_{ij2}{}^{U}, v_{ij3}{}^{U}, v_{ij4}{}^{U}; w_{ij}{}^{U})] = \tilde{\tilde{x}}_{ij} \otimes \tilde{\tilde{\omega}}_j$$

$$\left[\tilde{\tilde{v}}_{ij}\right]_{4\times 5} = \Bigg[\begin{matrix} \left[(0.689,0.735,0.908,0.946;0.800),(0.689,0.735,0.908,0.946;0.800)\right] \\ \left[(0.631,0.720,0.884,0.937;0.800),(0.631,0.720,0.884,0.937;1.000)\right] \\ \left[(0.787,0.882,0.968,0.988;0.800),(0.787,0.882,0.968,0.988;1.000)\right] \\ \left[(0.709,0.799,0.916,0.941;0.800),(0.709,0.799,0.916,0.941;1.000)\right] \end{matrix}$$

$$\begin{matrix} \left[(0.412,0.492,0.702,0.786;0.800),(0.412,0.492,0.702,0.786;1.000)\right] \\ \left[(0.560,0.643,0.831,0.898;0.800),(0.560,0.643,0.831,0.898,1.000)\right] \\ \left[(0.502,0.584,0.802,0.886;0.800),(0.502,0.584,0.802,0.886;1.000)\right] \\ \left[(0.341,0.427,0.624,0.707;0.800),(0.341,0.427,0.624,0.707;1.000)\right] \end{matrix}$$

$$\begin{matrix}[(0.487,0.572,0.792,0.874;0.800),(0.487,0.572,0.792,0.874;1.000)]\\ [(0.539,0.621,0.816,0.885;0.800),(0.539,0.621,0.816,0.885;1.000)]\\ [(0.452,0.527,0.744,0.835;0.800),(0.452,0.527,0.744,0.835;1.000)]\\ [(0.312,0.318,0.614,0.705;0.800),(0.312,0.318,0.614,0.705;1.000)]\end{matrix}$$

$$\begin{matrix}[(0.716,0.810,0.937,0.976;0.800),(0.716,0.810,0.937,0.976;1.000)]\\ [(0.698,0.788,0.910,0.947;0.800),(0.698,0.788,0.910,0.947;1.000)]\\ [(0.733,0.828,0.945,0.982;0.800),(0.733,0.828,0.945,0.982;1.000)]\\ [(0.527,0.623,0.782,0.844;0.800),(0.527,0.623,0.782,0.844;1.000)]\end{matrix}$$

$$\left.\begin{matrix}[(0.394,0.488,0.668,0.734;0.800),(0.394,0.488,0.668,0.734;1.000)]\\ [(0.264,0.336,0.558,0.655;0.800),(0.264,0.336,0.558,0.655;1.000)]\\ [(0.287,0.366,0.591,0.688;0.800),(0.287,0.366,0.591,0.688;1.000)]\\ [(0.237,0.311,0.553,0.617;0.800),(0.237,0.311,0.553,0.617;1.000)]\end{matrix}\right]$$

$$\begin{aligned}\tilde{\tilde{v}}^{+}{}_{1\times 5} = (&[(0.787,0.882,0.968,0.988;0.800),(0.787,0.882,0.968,0.988;1.000)]\\ &[(0.551,0.634,0.825,0.991;0.800),(0.551,0.634,0.825,0.991;1.000)]\\ &[(0.539,0.621,0.816,0.885;0.800),(0.539,0.621,0.816,0.885;1.000)]\\ &[(0.773,0.828,0.945,0.982;0.800),(0.773,0.828,0.945,0.982;1.000)]\\ &[(0.394,0.488,0.668,0.734;0.800),(0.394,0.488,0.668,0.734;1.000)])\end{aligned}$$

$$\begin{aligned}\tilde{\tilde{v}}^{-}{}_{1\times 5} = (&[(0.631,0.720,0.884,0.937;0.800),(0.631,0.720,0.884,0.937;1.000)]\\ &[(0.341,0.427,0.624,0.707;0.800),(0.341,0.427,0.624,0.707;1.000)]\\ &[(0.312,0.318,0.614,0.705;0.800),(0.312,0.318,0.614,0.705;1.000)]\\ &[(0.527,0.623,0.782,0.844;0.800),(0.527,0.623,0.782,0.844;1.000)]\\ &[(0.237,0.311,0.553,0.617;0.800),(0.237,0.311,0.553,0.617;1.000)])\end{aligned}$$

5. Calculate the weighted matrix and the COG of each attributes with respect to the positive ideal solution and the negative ideal solution (y,x):

$$y_{\tilde{a}} = \begin{cases} \dfrac{w_{\tilde{a}} \times \left(\dfrac{a_3 - a_2}{a_4 - a_1} + 2\right)}{6} & if\ a_1 \neq a_4 \\ w_{\tilde{a}} / 2 & if\ a_1 = a_4 \end{cases}$$

$$x_{\tilde{a}} = \frac{y_{\tilde{a}} \times (a_2 + a_3) + (a_1 + a_4) \times (w_{\tilde{a}} - y_{\tilde{a}})}{2 \times w_{\tilde{a}}}$$

$$[(y,x)_v]_{4\times 5} = \left[\begin{matrix}[(0.3336,0.8283),(0.4170,0.8283)],[(0.3415,0.5981),(0.4269,0.5981)],[(0.3425,0.6811),\\ [(0.3381,0.7916),(0.4227,0.7916)],[(0.3245,0.7542),(0.4057,0.7542)],[(0.3418,0.7148),\\ [(0.3237,0.9027),(0.4046,0.9027)],[(0.3424,0.6936),(0.4280,0.6936)],[(0.3422,0.6401),\\ [(0.3339,0.8386),(0.4174,0.8386)],[(0.3384,0.5246),(0.4230,0.5246)],[(0.3671,0.4890),\end{matrix}\right.$$

$$\left.\begin{matrix}(0.4281,0.6811)],[(0.3318,0.8574),(0.4147,0.8574)],[(0.3373,0.5699),(0.4216,0.5699)]\\ (0.4273,0.7148)],[(0.3320,0.8335),(0,4150,0.8335)],[(0,3424,0.4542),(0.4280,0.4542)]\\ (0.4278,0.6401)],[(0.3293,0.8694),(0.4116,0.8694)],[(0.3415,0.4837),(0.4268,0.4837)]\\ (0.4589,0.4890)],[(0.3335,0.6926),(0.4169,0.6926)],[(0.3516,0.4292),(0.4395,0.4292)]\end{matrix}\right]$$

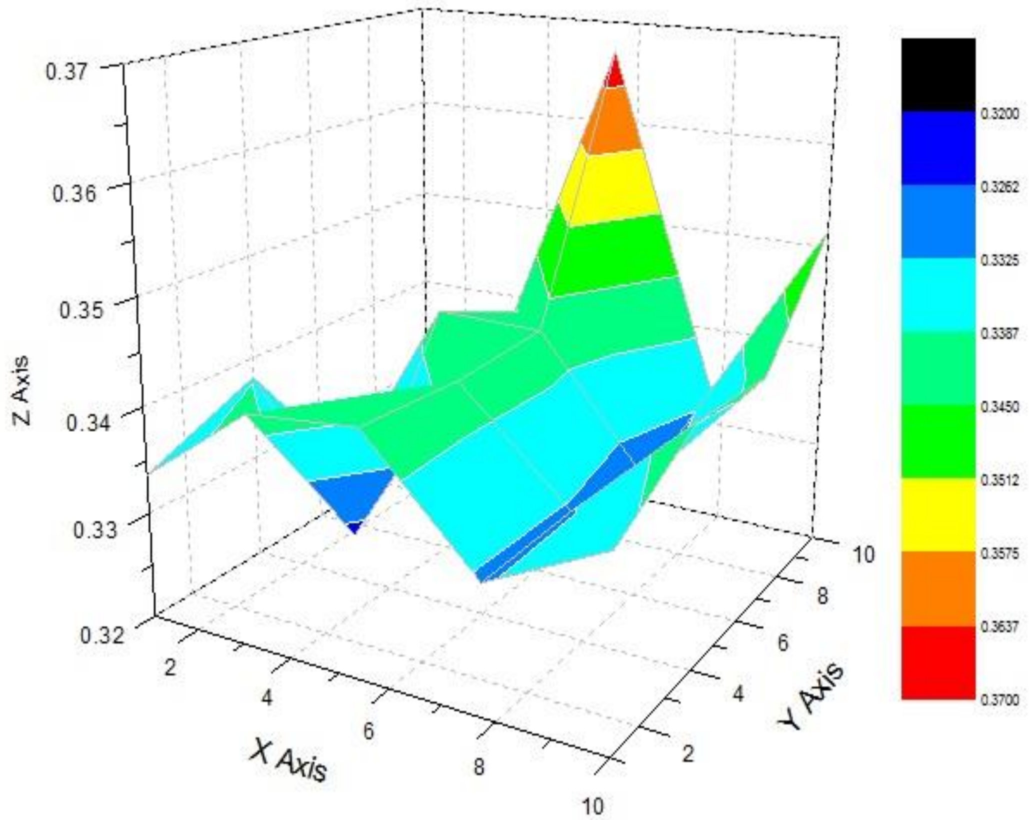

Fig-5: Three dimensional representation of the weighted decision matrix

$$[(y,x)_{v^+}]_{1\times 5} = [[(0.3237, 0.9027), (0.4046, 0.9027)], [(0.3245, 0.7542), (0.4057, 0.7542)], [(0.3418, 0.7148), (0.4273, 0.7148)], [(0.3293, 0.8694), (0.416, 0.8694)], [(0.3373, 0.5699), (0.4216, 0.5699)]]$$

$$[(y,x)_{v^-}]_{1\times 5} = [[0.3318, 0.7916), (0.4227, 0.7916)], [(0.3424, 0.5246), (0.4280, 0.5246)], [(0.3671, 0.4890), (0.4589, 0.4890)], [(0.3335, 0.6926), (0.4169, 0.6926)], [(0.3516, 0.4292), (0.4395, 0.4292)]]$$

5. Compute the values Si and Ri, i=1,2,3,4:

$$d(\tilde{\tilde{v}}_{j^+}, \tilde{\tilde{v}}_{ij}) = \sqrt{\frac{(y_{\tilde{\tilde{A}}^L} - y_{\tilde{\tilde{B}}^L})^2 + (x_{\tilde{\tilde{A}}^L} - x_{\tilde{\tilde{B}}^L})^2 + (y_{\tilde{\tilde{A}}^U} - y_{\tilde{\tilde{B}}^U})^2 + (x_{\tilde{\tilde{A}}^U} - x_{\tilde{\tilde{B}}^U})^2}{4}}$$

$$s_i = \sum_{j=1}^{n} \left[\frac{d(\tilde{\tilde{v}}_{j^+}, \tilde{\tilde{v}}_{ij})}{d(\tilde{\tilde{v}}_{j^+}, \tilde{\tilde{v}}_{j^-})} \right]$$

Hence

$$d(\tilde{\tilde{v}}_{j^+}, \tilde{\tilde{v}}_{ij}) = \begin{bmatrix} (0.0532, 0.1112, 0.0238, 0.0087, 0.0000) \\ (0.0794, 0.0000, 0.0000, 0.0255, 0.0519) \\ (0.0000, 0.0452, 0.0528, 0.0000, 0.0610) \\ (0.0461, 0.1627, 0.1609, 0.1251, 0.1001) \end{bmatrix},$$

$$d(\tilde{\tilde{v}}_{j^+}, \tilde{\tilde{v}}_{j^-}) = [(0.0794, 0.1630, 0.1609, 0.1251, 0.1001)]$$

$$S_1 = 1.5696,\ S_2 = 1.7225,\ S_3 = 1.2149,\ S_4 = 4.5875.$$

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Compute the values R_i $$R_i = \max_j \left[\frac{d(\tilde{\tilde{v}}_{j^+}, \tilde{\tilde{v}}_{ij})}{d(\tilde{\tilde{v}}_{j^+}, \tilde{\tilde{v}}_{j^-})} \right]$$

$R_1 = 0.6822$, $R_2 = 1.0000$, $R_3 = 0.6094$, $R_4 = 1.0087$

Compute the values Q_i, i=1,2,3,4

$$Q_i = v\frac{(S_i - S^*)}{(S^- - S^*)} + (1-v)\frac{(R_i - R^*)}{(R^- - R^*)}$$

where $S^* = \min_i S_i$, $S^- = \max_i S_i$, $R^* = \min_i R_i$, $R^- = \max_i R_i$

$S^* = 1.2149$, $S^- = 4.5875$, $R^* = 0.6094$, $R^- = 1.0087$

$Q_1 = 0.1438$, $Q_2 = 0.5626$, $Q_3 = 0.0000$, $Q_4 = 1.0000$

8. Rank the alternatives sorting by the value Q in decreased order the position in the front is better than in the behind. Hence we get:

$$a_3 \succ a_1 \succ a_2 \succ a_4.$$

From the ranking it can be seen that the best alternative is a_3.

METHOD 2:

From the previous procedure repeating step 1 to step 5, we get the following:

$$[(y,x)_v]_{4\times5} = \begin{bmatrix} [(0.3336,0.8283),(0.4170,0.8283)],[(0.3415,0.5981),(0.4269,0.5981)],[(0.3425,0.6811), \\ [(0.3381,0.7916),(0.4227,0.7916)],[(0.3245,0.7542),(0.4057,0.7542)],[(0.3418,0.7148), \\ [(0.3237,0.9027),(0.4046,0.9027)],[(0.3424,0.6936),(0.4280,0.6936)],[(0.3422,0.6401), \\ [(0.3339,0.8386),(0.4174,0.8386)],[(0.3384,0.5246),(0.4230,0.5246)],[(0.3671,0.4890), \end{bmatrix}$$

$$\begin{matrix} (0.4281,0.6811)],[(0.3318,0.8574),(0.4147,0.8574)],[(0.3373,0.5699),(0.4216,0.5699)] \\ (0.4273,0.7148)],[(0.3320,0.8335),(0,4150,0.8335)],[(0,3424,0.4542),(0.4280,0.4542)] \\ (0.4278,0.6401)],[(0.3293,0.8694),(0.4116,0.8694)],[(0.3415,0.4837),(0.4268,0.4837)] \\ (0.4589,0.4890)],[(0.3335,0.6926),(0.4169,0.6926)],[(0.3516,0.4292),(0.4395,0.4292)] \end{matrix}$$

$$[(y,x)_{v^+}]_{1\times5} = [[(0.3237,0.9027),(0.4046,0.9027)],[(0.3245,0.7542),(0.4057,0.7542)], [(0.3418,0.7148),(0.4273,0.7148)],[(0.3293,0.8694), (0.416,0.8694)],[(0.3373,0.5699),(0.4216,0.5699)]]$$

$$[(y,x)_{v^-}]_{1\times5} = [[0.3318,0.7916),(0.4227,0.7916)],[(0.3424,0.5246),(0.4280,0.5246)], [(0.3671,0.4890),(0.4589,0.4890)],[(0.3335,0.6926), (0.4169,0.6926)],[(0.3516,0.4292),(0.4395,0.4292)]]$$

In this method we find the mean value from the above computations and use a different kind of distance function :

$$[\tilde{\tilde{V}}_{ij}]_{4\times5} = \begin{pmatrix} (0.5810,0.6227) & (0.4698,0.5125) & (0.5118,0.5546) & (0.5946,0.6361) & (0.4536,0.4958) \\ (0.5646,0.6072) & (0.5394,0.5800) & (0.5283,0.5711) & (0.5828,0.6243) & (0.3983,0.4411) \\ (0.6132,0.6537) & (0.5180,0.5608) & (0.4912,0.5340) & (0.5994,0.6405) & (0.4126,0.4553) \\ (0.5863,0.6280) & (0.4315,0.4738) & (0.4281,0.4740) & (0.5131,0.5548) & (0.3904,0.4344) \end{pmatrix}$$

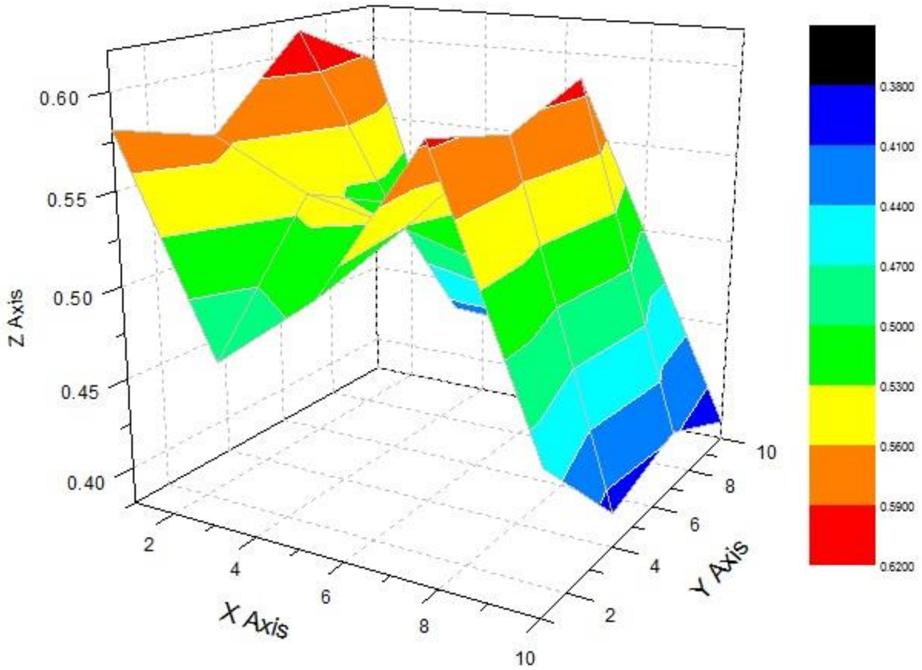

Fig-6: Three dimensional representation of the combined weighted decision matrix

$$\left[\tilde{\tilde{V}}_j^{\,+}\right]=\left[(0.6132,0.6537)\,(0.5394,0.5800)\,(0.5283,0.5711)\,(0.5994,0.6405)\,(0.4536,0.4958)\right]$$

$$\left[\tilde{\tilde{V}}_j^{\,-}\right]=\left[(0.5649,0.6072)\,(0.4335,0.4763)\,(0.4281,0.4740)\,(0.5131,0.5548)\,(0.3904,0.4344)\right]$$

6 . Calculate the values S_i and R_i , i = 1,2,3,4, using the following distance function:

$$d\left(\tilde{\tilde{A}},\tilde{\tilde{B}}\right)=\sqrt{\frac{(y_A-y_B)^2+(x_A-x_B)^2}{2}}$$

$$d\left(\tilde{\tilde{V}}_1^{+},\tilde{\tilde{V}}_{ij}\right)=\left[(0.0316\ ,\ 0.0686\ ,\ 0.0173\ ,\ 0.0046\ ,0\ )\right]$$

$$d\left(\tilde{\tilde{V}}_2^{+},\tilde{\tilde{V}}_{ij}\right)=\left[(\ 0.0476\ ,\ 0\ \ ,\ 0\ \ ,\ 0.0173\ ,\ 0.0550\ )\right]$$

$$d\left(\tilde{\tilde{V}}_3^{+},\tilde{\tilde{V}}_{ij}\right)=\left[(\ 0\ \ ,\ 0.0203\ ,\ 0.0262\ ,\ 0\ \ ,\ 0.0408\ )\right]$$

$$d\left(\tilde{\tilde{V}}_4^{+},\tilde{\tilde{V}}_{ij}\right)=\left[(\ 0.0263\ ,\ 0.1071\ ,\ 0.0987\ ,\ 0.0860\ ,\ 0.0623\ )\right]$$

$$d\left(\tilde{\tilde{V}}_j^{+},\tilde{\tilde{V}}_{ij}\right)=\begin{pmatrix}\left[(\ 0.0316\ ,\ 0.0686\ ,\ 0.0173\ ,\ 0.0046\ ,\ 0.0000\ )\right],\ \left[(\ 0.0476\ ,\ 0.0000\ ,\ 0.0000\ ,\ 0.0173\ ,\ 0.0550\ )\right], \\ \left[(\ 0.0000\ ,\ 0.0203\ ,\ 0.0262\ ,\ 0.0000\ ,\ 0.0408\ )\right],\ \left[(\ 0.0263\ ,\ 0.1071\ ,\ 0.0987\ ,\ 0.0860\ ,\ 0.0623\ )\right]\end{pmatrix}=$$

Calculate $d\left(\tilde{\tilde{V}}_j^{\,+},\tilde{\tilde{V}}_j^{\,-}\right)$:

$d\left(\tilde{\tilde{V}}_1^{+},\tilde{\tilde{V}}_1^{-}\right)$ = 0.0474, $d\left(\tilde{\tilde{V}}_2^{+},\tilde{\tilde{V}}_2^{-}\right)$ = 0.1048, $d\left(\tilde{\tilde{V}}_3^{+},\tilde{\tilde{V}}_3^{-}\right)$ = 0.0987

$d\left(\tilde{\tilde{V}}_4^{+},\tilde{\tilde{V}}_4^{-}\right)$ = 0.0860, $d\left(\tilde{\tilde{V}}_5^{+},\tilde{\tilde{V}}_5^{-}\right)$ = 0.0623

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$$d\left(\tilde{\tilde{V}}_j^{+}, \tilde{\tilde{V}}_j^{-}\right) = \left[\left(0.0474 \,,\, 0.1048 \,,\, 0.0987 \,,\, 0.0860 \,,\, 0.0623 \right)\right]$$

$$\mathbf{S_i} = \sum_{i=1}^{n} \left[\frac{d\left(\tilde{\tilde{V}}_j^{+}, \tilde{\tilde{V}}_{ij}\right)}{d\left(\tilde{\tilde{V}}_j^{+}, \tilde{\tilde{V}}_j^{-}\right)} \right]$$

$S_1 = 1.5500$; $S_2 = 2.0882$; $S_3 = 1.1141$; $S_4 = 4.5768$

Compute the value R_i ,

$$R_i = \max_j \left[\frac{d\left(\tilde{\tilde{V}}_j^{+}, \tilde{\tilde{V}}_{ij}\right)}{d\left(\tilde{\tilde{V}}_j^{+}, \tilde{\tilde{V}}_j^{-}\right)} \right]$$

$R_1 = 0.6667$, $R_2 = 1.0042$, $R_3 = 0.6549$, $R_4 = 1.0219$

7 . compute the value of Q_i , $i = 1, 2, 3, 4$

$$Q_i = v \left(\frac{S_i - S^*}{S^- - S^*} \right) + (1-v) \left(\frac{R_i - R^*}{R^- - R^*} \right)$$

Where , $S^* = \min_i s_i$, $R^* = \min_i R_i$,

$S^- = \max_i s_i$, $R^- = \max_i R_i$

$S^* = 1.1141$, $R^* = 0.6549$

$S^- = 4.5768$, $R^- = 1.0219$.

$Q_1 = 0.0789$, $Q_2 = 0.6165$, $Q_3 = 0$, $Q_4 = 1.000$

8) rank of the alternatives . sorting by the value Q in decreased order , the position in the front is better then in the behind. We get

$$a_3 \succ a_1 \succ a_2 \succ a_4$$

Hence a_3 is the best alternative.

METHOD 3:

From the previous procedure repeating step 1 to step 5, we get the following:

$$[(y,x)_v]_{4\times5} = \begin{bmatrix} [(0.3336, 0.8283), (0.4170, 0.8283)], [(0.3415, 0.5981), (0.4269, 0.5981)], [(0.3425, 0.6811), (0.4281, 0.6811)], [(0.3318, 0.8574), (0.4147, 0.8574)], [(0.3373, 0.5699), (0.4216, 0.5699)] \\ [(0.3381, 0.7916), (0.4227, 0.7916)], [(0.3245, 0.7542), (0.4057, 0.7542)], [(0.3418, 0.7148), (0.4273, 0.7148)], [(0.3320, 0.8335), (0,4150, 0.8335)], [(0,3424, 0.4542), (0.4280, 0.4542)] \\ [(0.3237, 0.9027), (0.4046, 0.9027)], [(0.3424, 0.6936), (0.4280, 0.6936)], [(0.3422, 0.6401), (0.4278, 0.6401)], [(0.3293, 0.8694), (0.4116, 0.8694)], [(0.3415, 0.4837), (0.4268, 0.4837)] \\ [(0.3339, 0.8386), (0.4174, 0.8386)], [(0.3384, 0.5246), (0.4230, 0.5246)], [(0.3671, 0.4890), (0.4589, 0.4890)], [(0.3335, 0.6926), (0.4169, 0.6926)], [(0.3516, 0.4292), (0.4395, 0.4292)] \end{bmatrix}$$

$$[(y,x)_{v^+}]_{1\times5} = \big([(0.3237, 0.9027), (0.4046, 0.9027)], [(0.3245, 0.7542), (0.4057, 0.7542)], [(0.3418, 0.7148), (0.4273, 0.7148)], [(0.3293, 0.8694), (0.416, 0.8694)], [(0.3373, 0.5699), (0.4216, 0.5699)]\big)$$

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$$[(y,x)_{v^-}]_{1\times 5} = ([0.3318, 0.7916), (0.4227, 0.7916)], [(0.3424, 0.5246), (0.4280, 0.5246)], [(0.3671, 0.4890), (0.4589, 0.4890)], [(0.3335, 0.6926), (0.4169, 0.6926)], [(0.3516, 0.4292), (0.4395, 0.4292)])$$

Calculating the mean value from the above computations and use the Hamming distance function we have the following computations :

$$[\tilde{\tilde{V}}_{ij}]_{4\times 5} = \begin{pmatrix} (0.5810, 0.6227) & (0.4698, 0.5125) & (0.5118, 0.5546) & (0.5946, 0.6361) & (0.4536, 0.4958) \\ (0.5646, 0.6072) & (0.5394, 0.5800) & (0.5283, 0.5711) & (0.5828, 0.6243) & (0.3983, 0.4411) \\ (0.6132, 0.6537) & (0.5180, 0.5608) & (0.4912, 0.5340) & (0.5994, 0.6405) & (0.4126, 0.4553) \\ (0.5863, 0.6280) & (0.4315, 0.4738) & (0.4281, 0.4740) & (0.5131, 0.5548) & (0.3904, 0.4344) \end{pmatrix}$$

$$\left[\tilde{\tilde{V}}_j^{\,+}\right] = \left[(0.6132, 0.6537)\ (0.5394, 0.5800)\ (0.5283, 0.5711)\ (0.5994, 0.6405)\ (0.4536, 0.4958)\right]$$

$$\left[\tilde{\tilde{V}}_j^{\,-}\right] = \left[(0.5649, 0.6072)\ (0.4335, 0.4763)\ (0.4281, 0.4740)\ (0.5131, 0.5548)\ (0.3904, 0.4344)\right]$$

The hamming distance is given by

$$d\left(\tilde{\tilde{V}}_j^{\,+}, \tilde{\tilde{V}}_{ij}\right) = \frac{1}{2}\sum_{i=1}^{n}\left\{ \left| y_A(x_i) - y_B(x_i)\right| + \left| x_A(x_i) - x_B(x_i)\right| \right\}$$

$$d\left(\tilde{\tilde{V}}_1^{\,+}, \tilde{\tilde{V}}_{ij}\right) = 0.1212,\ d\left(\tilde{\tilde{V}}_2^{\,+}, \tilde{\tilde{V}}_{ij}\right) = 0.1189,\ d\left(\tilde{\tilde{V}}_3^{\,+}, \tilde{\tilde{V}}_{ij}\right) = 0.09815,\ d\left(\tilde{\tilde{V}}_4^{\,+}, \tilde{\tilde{V}}_{ij}\right) = 0.3803,\ d\left(\tilde{\tilde{V}}_5^{\,+}, \tilde{\tilde{V}}_j^{\,-}\right) = 0.3966.$$

Calculate S_i :

$$S_1 = \frac{0.1212}{0.3966} = 0.3055,\ S_2 = \frac{0.1189}{0.3966} = 0.2994,\ S_3 = \frac{0.0981}{0.3966} = 0.2479,\ S_4 = \frac{0.3803}{0.3966} = 0.9589.$$

To find R_i ,

$$d\left(\tilde{\tilde{V}}_j^{\,+}, \tilde{\tilde{V}}_{ij}\right) = \max\left\{ \left| y_A(x_i) - y_B(x_i)\right| + \left| x_A(x_i) - x_B(x_i)\right| \right\}$$

$$d\left(\tilde{\tilde{V}}_1^{\,+}, \tilde{\tilde{V}}_{ij}\right) = 0.1317,\ d\left(\tilde{\tilde{V}}_2^{\,+}, \tilde{\tilde{V}}_{ij}\right) = 0.1100,\ d\left(\tilde{\tilde{V}}_3^{\,+}, \tilde{\tilde{V}}_{ij}\right) = 0.0815,\ d\left(\tilde{\tilde{V}}_4^{\,+}, \tilde{\tilde{V}}_{ij}\right) = 0.2141,\ d\left(\tilde{\tilde{V}}_5^{\,+}, \tilde{\tilde{V}}_j^{\,-}\right) = 0.2096.$$

The values of R_i, ,

$$R_1 = \frac{0.1371}{0.2096} = 0.6541,\quad R_2 = \frac{0.1100}{0.2096} = 0.5248,\quad R_3 = \frac{0.0815}{0.2096} = 0.3888,\quad R_4 = \frac{0.2141}{0.2096} = 1.0214.$$

To find Q_i :

$$Q_i = \alpha S_i + (1-\alpha) R_i$$

$$Q_1 = 0.4797,\ Q_2 = 0.4122,\ Q_3 = 0.3183,\ Q_4 = 0.9901$$

8) rank of the alternatives . sorting by the value Q in decreased order , the position in the front is better than in the behind. We get

$$a_3 \succ a_2 \succ a_1 \succ a_4$$

Hence a_3 is the best alternative.

Table.6 Comparison of the three different methods of Distance function.

METHOD	RANKING OF ALTERNATIVES
Method-1: Using distance function for Generalized interval valued trapezoidal fuzzy numbers $d(\tilde{\tilde{v}}_{j^+},\tilde{\tilde{v}}_{ij})=\sqrt{\frac{(y_{\tilde{\tilde{A}}^L}-y_{\tilde{\tilde{B}}^L})^2+(x_{\tilde{\tilde{A}}^L}-x_{\tilde{\tilde{B}}^L})^2+(y_{\tilde{\tilde{A}}^U}-y_{\tilde{\tilde{B}}^U})^2+(x_{\tilde{\tilde{A}}^U}-x_{\tilde{\tilde{B}}^U})^2}{4}}$	$a_3 \succ a_1 \succ a_2 \succ a_4$
Method-2: Using distance function for Generalized interval valued trapezoidal fuzzy numbers $d\left(\tilde{\tilde{A}},\tilde{\tilde{B}}\right)=\sqrt{\frac{(y_A-y_B)^2+(x_A-x_B)^2}{2}}$	$a_3 \succ a_1 \succ a_2 \succ a_4$
Method-3: Using distance function for Generalized interval valued trapezoidal fuzzy numbers $d\left(\tilde{\tilde{A}},\tilde{\tilde{B}}\right)=\frac{1}{2}\sum_{i=1}^{n}\left\{ \left\lvert y_A(x_i)-y_B(x_i)\right\rvert + \left\lvert x_A(x_i)-x_B(x_i)\right\rvert \right\}$	$a_3 \succ a_2 \succ a_1 \succ a_4$

From the table it can be observed that, the ranking of the best alternative a_3 has not changed in all the three different methods, but a variation can be observed only in the ranking of the alternatives a_1 and a_2 , which does not affect the decision making in general.

IV. CONCLUSION

The traditional VIKOR method is generally suitable for the decision making information taking the form of numerical values, and yet it will fail in dealing with the generalized interval-valued trapezoidal fuzzy numbers. In this work, with respect to Multiple Attribute Group Decision Making (MAGDM) problems in which the attribute weights and attribute values take the form of the generalized interval-valued trapezoidal fuzzy numbers, a new group decision making analysis method is discussed. An extended VIKOR method is presented to solve the MAGDM problems in which the attribute weights and values are given with the form of GIVTFN. Finally, an illustrative example has been given to show the steps of the developed method. It shows that this method is simple and easy to understand and it constantly enriches and develops the theory and method of MAGDM, and proposed a new idea for solving the MAGDM problems. Three different methods are used to show that the final decision remains unaltered even when different distance functions are used.

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