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A Primer on the TOPSIS-DMSS Technique with Entropy Method

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Abstract: **In this work scientific and simple calculation method for manufacturer's decision-makers to choose the most ideal supplier has been provided. This paper deals with the supplier selection problem based on TOPSIS algorithm (Technique for Order Preference by Similarity to Ideal Solution) with the application of entropy method which is a multiple criteria decision making approach and a Decision Making Support System (DMSS) technique. The TOPSIS algorithm deals with the conflicts between indicators based on certain way to sort the scheme and choose the best scheme. A numerical example is proposed to illustrate the effectiveness of this algorithm. However, Sensitivity Analysis for the weighting vectors is performed to make the result of evaluations more objective and accurate.**

Keywords: **TOPSIS, Decision Making Support Systems, MAGDM, Entropy, Attribute Weights, Ideal Solution.**

I. INTRODUCTION

The TOPSIS method was first developed by Hwang & Yoon [5] and ranks the alternatives according to their distances from the positive ideal and the negative ideal solution, i.e. the best alternative has simultaneously the shortest distance from the ideal solution and the farthest distance from the negative ideal solution. The ideal solution is identified with a hypothetical alternative that has the best values for all considered criteria whereas the negative ideal solution is identified with a hypothetical alternative that has the worst criteria values. In practice, TOPSIS has been successfully applied to solve selection/evaluation problems with a finite number of alternatives [2, 10, 11, 12, 14, 15] because it is intuitive and easy to understand and implement. Furthermore, TOPSIS has a sound logic that represents the rationale of human choice and has been proved to be one of the best methods in addressing the issue of rank reversal. In multiple attribute decision making (MADM) problem, a decision maker (DM) has to choose the best alternative that satisfies the evaluation criteria among a set of candidate solutions [1, 3, 4, 8, 13]. It is generally hard to find an alternative that meets all the criteria simultaneously, so a better solution is preferred. The TOPSIS method was developed for multi-criteria optimization of complex systems. This method focuses on ranking and selecting from a set of alternatives in the presence of conflicting criteria. Multi-criteria optimization is the process of determining the best feasible solution according to the established criteria (representing different effects). Practical problems are often characterized by several non-commensurable and conflicting criteria and there may be no solution satisfying all criteria simultaneously. In this paper while calculating the relative distance of each alternative to the positive ideal and negative ideal or anti-ideal solution, the sensitivity analysis is also performed together with a method of entropy. The final ranking of alternatives obtained from the proposed three methods is compared.

II. APPLICATION OF TOPSIS AS A DECISION MAKING SUPPORT SYSTEM (DMSS) TECHNIQUE

Decision support system (DSS) is seen as building blocks that offers the best combination of computational power, value for money and significantly offers efficiency in certain decision making problem solving [3, 8, 9, 13]. Based on these building blocks, modern DSS applications comprise of integrated resources working together which are model base, database or knowledge base, algorithms, user interface and control mechanisms used to support certain decision problem. Janic [7] stated that the TOPSIS method embraces seven steps which are as follows: (1) constructing the normalized decision matrix by using the decision making matrix; (2) constructing the weighted-normalized decision matrix; (3) determining the positive ideal and negative ideal solution; (4) calculating the separation measure of each alternative from the ideal one; (5) calculating the relative distance of each alternative to the ideal and negative ideal solution; (6) ranking the alternatives in descending order with respect to relative distance to the ideal solution;(7) identification of the preferable alternative as the closest to the ideal solution. However, in considering group decision making problems, the preferences among alternatives have to be aggregated for individual decision makers. After ranking the alternatives by utilizing TOPSIS individually, the geometric and the arithmetic mean methods are often used to aggregate the separation measures, ideal solutions or relative distances from individual decision makers. Usually, the mean approaches are suitable under a condition in which the decision makers are of equal importance and a consensus is about to be reached. In a real-world situation, some decision makers may strongly prefer some particular alternatives, and in the meantime, some other decision makers may prefer none at all. In such a case, the mean aggregation may result in a dissatisfactory final decision for some decision makers. Huang et al. [6] states that an individual with greater preferential differences among alternatives would have more influence in a group than those who with less preferential differences, since in such a case, the individual with greater preferential differences would fight for his/her choices, while the other members may be less insistent because of their similar perceptions of all alternatives. The mean approaches may thus not be able to achieve a robust consensus, and a satisfactory level of commitment is not anticipated. In [6] was proposed two indices, preferential differences and preferential ranks, to deal with this problem. However, their research was designed for AHP, and they integrated these two indices by an additive approach. This paper formulates the Aggregation operators to the use of TOPSIS, and employs the logical thinking of TOPSIS to construct the proposed group decision making approach. TOPSIS logical thinking considers that the optimal decision should have the closest distance from the best alternative and the farthest distances from the worst alternative.

A. The TOPSIS Method:

Decision-making problem is the process of finding the best option from all of the feasible alternatives. In almost all such problems, the multiplicity of criteria for judging the alternatives is pervasive [5, 16]. For many such problems, the Decision Maker wants to solve a multiple attribute decision making (MADM) problem. A MADM problem can be concisely expressed in matrix format as:

$$\begin{array}{cc} & C_n \\ A_m & \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \cdots & \cdots & & \cdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \end{array}$$

where $A_1, A_2, \ldots, A_m$ are possible alternatives among which decision makers have to choose, $C_1, C_2, \ldots, C_n$ are criteria with which alternative performance are measured, x_{ij} is the rating of alternative A_i with respect to criterion C_j. In a MADM model, the ideal solution such A* is the one that has the greatest utility on all of the attributes that is $A^*=\{c_1^*, c_2^*, c_3^*, \ldots, c_k^*\}$; $c_j^* = \max_i u_j(r_{ij})$, i=1,2,…,m, j=1,2,…,k. And the worst or the anti –ideal alternative such A is the one that has the least utility on all the attributes. That is $A^-=\{c_1^-, c_2^-, c_3^-, \ldots, c_k^-\}$, $c_j^- = \min_i u_j(r_{ij})$, i=1,2,…,m, j=1,2,…,k.

The TOPSIS technique by considering the difference of alternatives from ideal and anti-ideal solution, selects the one that has least difference from ideal and the greatest difference from the anti –ideal solution. TOPSIS technique has the following steps for solving MADM models.

STEPS 1: Transform decision making matrix to a normalized matrix by using the Euclidean norm given as;

$$r_{ij} = \frac{d_{ij}}{\sqrt{\sum_{i=1}^{m} d_{ij}^2}}$$

NOTE: If there are qualitative attributes , we can use scales for quantifying them in order to solve by TOPSIS technique.

STEPS 2: Calculate weighted normalized matrix $v=(v_{ij})_{mxk}$ by considering the normalized matrix from step1 and the vector of attributes weights from Decision Maker(DM) that is mxk matrix and its elements are $v_{ij} = r_{ij}.w_j$, i=1,2,…,m, j=1,2,…,k.

STEPS 3: Determine the ideal and anti-ideal solution by considering the weighted normalized matrix as:

$$A^{+} = \left\{ \left(max_i v_{ij} \,/\, j \in J \right), \left(min_i v_{ij} \,/\, j \in J' \right), / \, i = 1,2,\ldots,m \right\}$$

$$= \left\{ v_1^{+}, v_2^{+}, \ldots, v_j^{+}, \ldots, v_k^{+} \right\}$$

$$A^{-} = \left\{ \left(min_i v_{ij} \,/\, j \in J \right), \left(max_i v_{ij} \,/\, j \in J' \right), / \, i = 1,2,\ldots,m \right\}$$

$$= \left\{ v_1^{-}, v_2^{-}, \ldots, v_j^{-}, \ldots, v_k^{-} \right\}$$

STEPS 4: Calculate the distance of alternatives from ideal and anti-ideal solution.

For this, usually the Euclidean norm is used as follows:

$$d_i^{+} = \{\Sigma (v_{ij} - v_j^{+})^2\}^{(1/2)}, \; d_i^{-} = \{\Sigma (v_{ij} - v_j^{-})^2\}^{(1/2)}$$

Wherein, d_i^{+} is the distance of the i^{th} alternatives from the ideal solution and d_i^{-} is that of anti-ideal solution.

STEPS 5: Calculate the relative distance of alternatives A_i from ideal solution as:

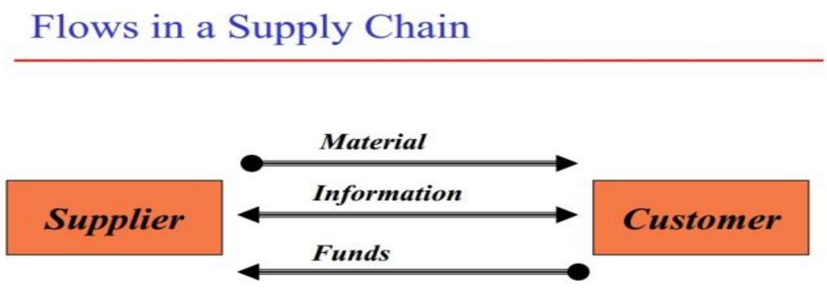
$$cl_i^{+} = \frac{d_i^{-}}{d_i^{-} + d_i^{+}}, \; i = 1,2,3.$$

Then, sort them by cl_i^{+} descending.

III. SUPPLIER SELECTION PROBLEM WITH TOPSIS METHOD AND SENSITIVITY ANALYSIS

Supplier Selection Model (SCM) emphasizes on the strategic cooperative relationship between core enterprise and enterprise alliance. SCM includes managing supply and demand, sourcing raw materials and parts, manufacturing and assembly, warehousing and inventory tracking, order entry and order management, distribution across all channels, and delivery to the customer. Under the environment of globalization market competition and cooperation, SCM is an effective model of enterprise operation and management. As enterprises pay more and more attention to their core competence they are increasingly unwilling to devote the capital, time and energy to those businesses which they are not familiar with and not good at. This change is also reflected in supply system, i.e., enterprise outsourcing or seeking for proper suppliers who can provide the businesses or services that are provided by the enterprise itself in the past.

Under the integrated SCM environment for some specific goals and benefits enterprises incline to form the strategic cooperative relationship. It is beneficial to each side and it is favorable to reduce total cost, decrease the storage, enhance information sharing, improve mutual communication, keep the consistent partnership, and create better competitive advantages. Thus at every node of supply chain the finance in the situation, quality, production, customer satisfaction and performance can be enhanced and improved. Of course, strategic cooperation requires the emphasis on cooperation and confidence. Some failures of operation and management in enterprises result either from instability of core enterprise or from instability of suppliers. In order to reduce the cost and risk of SCM, enterprises should make sound decisions on supplier selection and share benefits with them. Supplier management should include supplier's credit and reputation, product price, quality, delivery date etc. Supplier, as the object of enterprise purchasing activities, directly determines the quality of the raw materials and parts purchased by the core enterprise, and the supplier greatly influences the competitive competence of the product produced by the core enterprise. Therefore, a good decision-making method of supplier selection is quite necessary.



The flows resemble a chain reaction.

Several criteria have been identified for supplier selection, such as supplier's credit and reputation, product price, delivery date, the net price, quality, capacity and communication systems, historical supplier performance and so forth. Supplier, as the object of enterprise purchasing activities, it directly determines the quality of the raw material and parts purchased by the manufacturer, and the supplier selection is one of the essential steps in supply chain design. Since selecting the right suppliers considerably shrinks the purchasing cost and improves competitiveness, the supplier selection process is known as the most significant act of a purchasing department. Furthermore, a good decision-making method of supplier selection is quite necessary. So in this work, we use TOPSIS algorithm with entropy method to select suppliers effectively.

Earlier researches on the sensitivity analysis of MADM problems often focused on finding the least value of the change. However a new method for sensitivity analysis of MADM problems is considered in this article that calculates the changing in the finals score of alternatives when a change occurs in the weight of one attribute.

The vector for weights of attributes is $w^t=(w_1, w_2, \ldots\ldots W_k)$ wherein weights are normalized with a sum of 1 which is given as $\sum_{j=1}^{k} w_j = 1.$

With these assumptions, if the weight of other attributes change accordingly, then the new vector of weights is transformed into $w'^t = (w'_1, w'_2 \ldots w'_k)$.

The following theorem depicts changes in the weights of attributes:

THEOREM: In the, MADM model, if the weight of the p^th attributes, changes Δp, then the weight of other attributes change by Δ_j where, $\Delta_j = \frac{\Delta_p . w_j}{w_p - 1}; \quad j = 1, 2, \ldots, k, \; j \neq p$.

Proof: If new weights of attributes are w'_j, and new weights of p^th change is,

$$w_p' = w_p + \Delta_p \rightarrow (1)$$

Then, the new weight of the other attributes would change as

$$w_j' = w_j + \Delta_j; \quad j = 1, 2, \ldots, k, \; j \neq p \rightarrow (2)$$

The sum of the weight must be 1. Then,

$$\sum_{j=1}^{k} w_j' = \sum_{j=1}^{k} w_j + \sum_{j=1}^{k} \Delta_j$$

$$\sum_{j=1}^{k} w_j' - \sum_{j=1}^{k} w_j = \sum_{j=1}^{k} \Delta_j$$

$$\sum_{j=1}^{k} \Delta_j = 1-1$$

$$\boxed{\sum_{j=1}^{k} \Delta_j = 0} \rightarrow (3)$$

Here we have,

$$w_p' = w_p + \Delta_p \rightarrow (a)$$

$$\sum_{j=1}^{k} w_j' = \sum_{j=1}^{k} w_j + \sum_{j=1}^{k} \Delta_p, \; j = 1, 2, \ldots.k, \; j \neq p \rightarrow (b)$$

From (a)+(b) we have,

$$(w_p' - w_p) + \sum_{j=1}^{k} w_j + \sum_{j=1}^{k} w_j = \Delta_p + \sum_{j=1}^{k} \Delta_j$$

$$0 + 1 + 1 = \Delta_p + \sum_{j=1}^{k} \Delta_j$$

$$0 = \Delta_p + \sum_{j=1}^{k} \Delta_j$$

$$\boxed{\Delta_p = -\sum_{j=1}^{k} \Delta_j} \rightarrow (4)$$

Where,

$$\Delta_j = \frac{\Delta_p . w_j}{w_p - 1}; \qquad j = 1, 2, \ldots.k, \; j \neq p \rightarrow (5)$$

$$\Delta_p = -\sum_{j=1}^{k} \Delta_j \qquad \text{(from(4))}$$

since $$-\Delta_p = \sum_{j=1}^{k} \Delta_j = \sum_{j=1}^{k} \frac{\Delta_p . w_j}{w_p - 1} = \frac{\Delta_p}{w_p - 1} \sum_{j=1}^{k} w_j \rightarrow (A)$$

$$= \frac{\Delta_p}{w_p - 1}(1 - w_p)$$

$$= -\Delta_p \rightarrow (6)$$

Then,

$$\Delta_j = \frac{\Delta_p . w_j}{w_p - 1}$$

$$\Delta_j (w_p - 1) = \Delta_p . w_j$$

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$$\Delta_j(w_p - 1) = -\sum_{j=1}^{k} \Delta_j . w_j \qquad \text{(from(4))}$$

$$(w_p - 1) = -\sum_{j=1}^{k} w_j \rightarrow (6)$$

sub (6) in (A)

$$-\Delta_p = -\frac{\Delta_p}{(w_p - 1)}(1 - w_p) = -\frac{\Delta_p}{(w_p - 1)}\left\{-(w_p - 1)\right\} = -\Delta_p \rightarrow (7)$$

MAIN RESULT:

In a MADM problem, if the weight of the p[th] attribute changes from w_p to w_p' as:

$$w_p' = w_p + \Delta_p' \rightarrow (8)$$

Then the weight of other attribute would change as,

$$w_j' = w_j + \Delta_j = w_j + \frac{\Delta_p . w_j}{w_p - 1} = \frac{w_j\left(w_p - 1\right) + \Delta_p . w_j}{w_p - 1} = \frac{-(1 - w_p - \Delta_p).w_j}{-(w_p - 1)} = \frac{-(1 - w_p - \Delta_p).w_j}{-(w_p - 1)}$$

$$w_j' = \frac{(1 - w_p - \Delta_p).w_j}{(w_p - 1)} \rightarrow (9)$$

$$w_j' = \frac{(1 - w_p')}{(1 - w_p)} . w_j \rightarrow (10), \qquad j = 1, 2, ,, , k, j \neq p \qquad \text{(from(8))}$$

Then new vector for weights of attributes would be $, w'^t = (w'_1, w'_2 \ldots w'_k)$

$$w_j' = \begin{cases} w_j + \Delta_p, & \text{iff } j = p \\ \frac{(1 - w_p')}{(1 - w_p)} . w_j, & \text{iff } j \neq p, \ j = 1, 2, \ldots, k, \end{cases}$$

Here,

$$\Rightarrow w_p' = w_p + \Delta_p$$

$$\Rightarrow \begin{cases} \text{if } w_p' > w_p \quad \Rightarrow \quad w_j' < w_j \\ \text{if } w_p' < w_p \quad \Rightarrow \quad w_j' > w_j \qquad j \neq p, \quad j = 1, 2, \ldots, k \qquad \rightarrow (11) \end{cases}$$

The sum of new weights of attributes is:

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$$\therefore \sum_{j=1}^{k} W_j' = \sum_{j=1, j\neq p}^{k} w_j' + w_p' = \sum_{j=1, j\neq p}^{k} \frac{(1-w_P-\Delta_P)\, w_j}{(w_p-1)} + (w_p+\Delta_p) \qquad \text{(from(9) \& (10))}$$

$$= \frac{(1-w_p-\Delta_p)}{(w_p-1)}\left(\sum_{j=1, j\neq p}^{k} w_j\right) + w_p + \Delta_p$$

$$= \frac{(1-w_p-\Delta_p)}{(w_p-1)}(w_p-1) + w_p + \Delta_p \qquad \because \sum_{j=1, j\neq p}^{k} w_j = 1 - w_p$$

$$= 1 - w_p + w_p + \Delta_p$$

$$\therefore \boxed{\sum_{j=1}^{k} w_j' = 1} \rightarrow (12)$$

Corollary: In the new vector of weights that is obtained by (11) the weights ratio is the same (exception of the p^{th}attribute) because new weights for attributes (except the p^{th} attribute) is obtained by multiplying the constant $\frac{1-w_p-\Delta_p}{1-w_p}$ to the old weight .Then the ratio of new weight of attribute c_i to new weight of attribute c_j for i , j=1,2,…,k, j≠p is the same to ratio of old ones .That is

$$\frac{w_i'}{w_j'} = \frac{w_i}{w_j}, \qquad \text{i, j=1,2,…,k, j} \neq \text{p} \rightarrow (13)$$

In a decision making problem solved by TOPSIS if the weight of one attributes changes, then the final score of alternatives will change. The next theorem calculates the change.

Theorem: In the MADM model of TOPSIS , if the weight of the p^{th} attribute changes by Δp, the final score of the i^{th} alternative i=1,2,…,m would change as below;

$$cl_i^{+} = \frac{d_i'^{-}}{d_i'^{+} + d_i'^{-}} \rightarrow (14)$$

where $d_i'^{+}, d_i'^{-}$ are calculated as follows:-

$$d_i'^{+} = \{\gamma^2 . d_i^{+2} + (1-\gamma^2)(v_{ip} - v_p^{+})^2 + \Delta_p^2 (r_{ip} - r_{lp})^2 + 2\Delta_p (v_{ip} - v_p^{+})^2 (r_{ip} - r_{lp})^2\}^{(1/2)} \rightarrow (15)$$

$$d_i'^{-} = \{\gamma^2 . d_i^{-2} + (1-\gamma^2)(v_{ip} - v_p^{-})^2 + \Delta_p^2 (r_{ip} - r_{lp})^2 + 2\Delta_p (v_{ip} - v_p^{-})^2 (r_{ip} - r_{lp})^2\}^{(1/2)} \rightarrow (16)$$

For simplicity the following changes are performed:

$$\gamma = \frac{1-w_p-\Delta_p}{1-wp} = \frac{1-w_p'}{1-wp} \rightarrow (17)$$

$$w_p' = w_p + \Delta_p \Rightarrow \begin{cases} if \quad 0 < \gamma < 1 \Rightarrow w_p' > w_p \\ if \quad \gamma > 1 \Rightarrow w_p' < w_p \end{cases} \rightarrow (18)$$

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$$l = \begin{cases} \max_i \; v_{ip} & if \;\; p \in J, \; i = 1,2,\ldots,m \\ \min_i \; v_{ip} & if \;\; p \in J', \; i = 1,2,\ldots,m \end{cases} \rightarrow (19)$$

$$l' = \begin{cases} \min_i \; v_{ip} & if \;\; p \in J, \; i = 1,2,\ldots,m \\ \max_i \; v_{ip} & if \;\; p \in J', \; i = 1,2,\ldots,m \end{cases} \rightarrow (20)$$

Proof: By considering equation (18),if the weight of the p[th] attribute changes by Δp ,then the weights of other attributes would change by:

$$w_j' = \frac{(1 - w_p - \Delta_p)}{1 - w_p} . w_j$$

$$w_j' = \frac{1 - w_p'}{1 - w_p} . w_j \qquad \left(\because \frac{1 - w_p'}{1 - w_p} = \gamma \right)$$

$$w_j' = \gamma . w_j, \qquad \rightarrow (21) \qquad (j = 1,2,\ldots,k, \; j \neq p)$$

To prove equation (15) & (16) we consider these changes in all steps of TOPSIS technique with regard to changes in the weights, the weighted normalized matrix $v = (v_{ij}')_{m \times k}$ as:

$$v_{ij}' = w_j' . r_{ij} \qquad \left(\because w_j' = \left(\frac{1 - w_p - \Delta_p}{1 - w_p} . w_j \right) \right)$$

$$v_{ij}' = \left(\frac{1 - w_p - \Delta_p}{1 - w_p} . w_j \right) r_{ij} \qquad \left(\because w_j . r_{ij} = \gamma_{ij} \right)$$

$$v_{ij}' = \frac{1 - w_p - \Delta_p}{1 - w_p} . w_j . r_{ij}, \qquad \rightarrow (22) \qquad (i = 1,2,\ldots,m, \; j = 1,2\ldots.k, \; j \neq p)$$

Since the ideal and anti-ideal solutions are calculated from weighted normalized decision matrix and in both $(j = p, \; j \neq p)$ the values of v_{ip}'s at each column changes. similarly, no changes would occur in calculating the ideal and anti-ideal solutions and only their value changes as follows:

If $j = p$ then,

$$\left. \begin{array}{l} v_p'^{+} = v_p^{+} + \Delta_p . r_{ip} \\ v_p'^{-} = v_p^{-} + \Delta_p . r_{ip} \end{array} \right\} \rightarrow (23)$$

where,

And if $j \neq p$ then,

$$V_j^{+'} = v_j + \left(\frac{(1-w_p - \Delta_p}{1-w_p} \right) = v_j + \left(\frac{1-w_p{'}}{1-w_p} \right)$$

$$V'_{j-} = v_j - \left(\frac{(1-w_p - \Delta_p)}{1-w_p} \right)$$

$$= v_j -. \left(\frac{1-w_p{'}}{1-w_p} \right)$$

$$V'_{j-} = \left(v_{j-} \right) .\gamma \qquad \rightarrow (24), \quad j=1,2,\ldots,k$$

By performing these changes, the distance of alternatives from the ideal anti-ideal solution would change as:

$$d_i^{'+} = \left\{ \sum_{j=1}^{k} ((v_{ij}{'} - v_j^{'+})^2)^{(1/2)} \right\}$$

$$= \left\{ \sum_{j=1 j\neq P}^{k} (v_{ij} - v_j^{'+})^2 .\gamma^2 + (v_{ip} + \Delta_p .r_{ip} - v_p^{+} - \Delta_p .r_{lp})^2 \}^{(1/2)} \right\} \rightarrow (25)$$

$$d_i^{'-} = \left\{ \sum_{j=1}^{k} ((v_{ij}{'} - v_j^{'-})^2)^{(1/2)} \right\}$$

$$= \left\{ \sum_{j=1 j\neq P}^{k} (v_{ij} - v_j^{'-})^2 .\gamma^2 + (v_{ip} + \Delta_p .r_{ip} - v_p^{-} - \Delta_p .r_{lp})^2 \}^{(1/2)} \right\} \rightarrow (26)$$

By solving and simplifying (25)&(26) equation (15)&(16) are acquired.

The values $d_i^{'+}, d_i^{'-}$ in equation (15)&(16) are calculated by change in the weight of the p^th attribute Δp, and other available information in the model.

IV. ILLUSTRATIVE EXAMPLE

We assume a MADM supplier selection problem that has three alternatives and four attributes where in attributes c_1, c_4 are of cost type and attribute c_2, c_3 are of profit type. Let the weight vector be given as $W^t=(0.4,0.2,0.3,0.1)$.

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{bmatrix} 13 & 9 & 9 & 8 \\ 5 & 3 & 5 & 12 \\ 7 & 5 & 7 & 6 \end{bmatrix} \end{array}$$

Method-1: TOPSIS Technique

For solving it by TOPSIS technique, normalized matrix by using Euclidean norm is calculated as:

$$r_{ij} = \frac{d_{ij}}{\sqrt{\sum_{i=1}^{m} d_{ij}^2}} \qquad i=1,2,3, \quad j=1,2,3,4.$$

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1) Calculate $\left(\sum d_{ij}^2\right)^{(1/2)}$ for each column and divide each column by the same to get r_{ij} .

$$d_{11}=\frac{13}{\sqrt{13^2+5^2+7^2}}=0.83;\quad d_{12}=\frac{9}{\sqrt{3^2+5^2+9^2}}=0.84;\quad d_{13}=\frac{9}{\sqrt{9^2+5^2+7^2}}=0.72;\quad d_{14}=\frac{8}{\sqrt{8^2+12^2+6^2}}=0.51;$$

$$d_{21}=\frac{5}{\sqrt{13^2+5^2+7^2}}=0.38;\quad d_{22}=\frac{3}{\sqrt{3^2+5^2+9^2}}=0.28;\quad d_{23}=\frac{5}{\sqrt{9^2+5^2+7^2}}=0.40;\quad d_{24}=\frac{12}{\sqrt{8^2+12^2+6^2}}=0.77;$$

$$d_{31}=\frac{7}{\sqrt{13^2+5^2+7^2}}=0.41;\quad d_{32}=\frac{5}{\sqrt{3^2+5^2+9^2}}=0.47;\quad d_{33}=\frac{7}{\sqrt{9^2+5^2+7^2}}=0.56;\quad d_{34}=\frac{8}{\sqrt{8^2+12^2+6^2}}=0.38.$$

$$R=\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array}\begin{array}{c} \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{bmatrix} 0.83 & 0.84 & 0.72 & 0.51 \\ 0.38 & 0.28 & 0.40 & 0.77 \\ 0.41 & 0.47 & 0.56 & 0.38 \end{bmatrix} \end{array}$$

From the equation $v_{ij}=r_{ij}.w_j$, i=1,2,,3 j=1,2,3,4 we get the weighted normalized matrix.

2)Multiply each column by $w_j$to get $v_{ij:}$

$$\bar{v}=\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array}\begin{array}{c} \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{bmatrix} 0.83*0.4 & 0.84*0.2 & 0.72*0.3 & 0.51*0.1 \\ 0.38*0.4 & 0.28*0.2 & 0.40*0.3 & 0.77*0.1 \\ 0.41*0.4 & 0.47*0.2 & 0.56*0.3 & 0.38*0.1 \end{bmatrix} \end{array}$$

$$\bar{v}=\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array}\begin{array}{c} \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{bmatrix} 0.33 & 0.17 & 0.22 & 0.05 \\ 0.15 & 0.06 & 0.12 & 0.08 \\ 0.17 & 0.09 & 0.17 & 0.04 \end{bmatrix} \end{array}$$

4)Determine ideal solution $A^{+:}$

Since $J=\{1,2\}, J=\{1,4\}$ then ,ideal and anti-ideal solutions would be:

$$A^{+}=\{0.15, 0.17, 0.22, 0.04\}$$

$$\bar{v}=\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array}\begin{array}{c} \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{bmatrix} 0.33 & 0.17 & 0.22 & 0.05 \\ 0.15\downarrow & 0.06\uparrow & 0.12\uparrow & 0.08\downarrow \\ 0.17 & 0.09 & 0.17 & 0.04 \end{bmatrix} \end{array}$$

5)Negative ideal solution A^{-} :

$$A^{-}=\{0.33, 0.06, 0.12, 0.08\}$$

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$$\bar{v} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{bmatrix} 0.33 & 0.17 & 0.22 & 0.05 \\ 0.15\uparrow & 0.06\downarrow & 0.12\downarrow & 0.08\uparrow \\ 0.17 & 0.09 & 0.17 & 0.04 \end{bmatrix} \end{array}$$

By using the Euclidean norm, distance of alternatives from ideal and anti -ideal solutions are:

$$d_i^+ = \{\sum(v_{ij} - v_j^+)^2\}^{(1/2)}, \; d_i^- = \{\sum(v_{ij} - v_j^-)^2\}^{(1/2)}$$

$$\boxed{d_1^+ = 0.18} \; \boxed{d_2^+ = 0.152} \; \boxed{d_3^+ = 0.09}$$

$$\boxed{d_1^- = 0.15} \; \boxed{d_2^- = 0.179} \; \boxed{d_3^- = 0.8}$$

Then the final score of alternatives are calculated by,

$$cl_i^+ = \frac{d_i^-}{d_i^- + d_i^+}, i = 1,2,3$$

$$cl_1^+ = \frac{d_1^-}{d_1^- + d_1^+} = \frac{0.15}{0.15 + 0.18} \qquad \boxed{cl_1^+ 0.454}$$

Similarly, the other values are calculated: $\boxed{cl_2^+ = 0.540}$ $\boxed{cl_3^+ = 0.893}$

Therefore, alternatives are ranked as: $\boxed{A_3 > A_2 > A_1.}$

Method-2: Sensitivity analysis to TOPSIS method

Now we assume that the weight of the 2nd attribute increased by $\Delta_2 = 0.2$ and be

$$w_2' = w_2 + \Delta_2 = 0.2 + 0.2 = 0.4$$

Then by the equations

$$w_j' = \frac{1 - w_2'}{1 - w_2}.w_j, \; j = 1,3,4$$ the weight of other attributes change as

$$w_j' = \frac{1 - 0.4}{1 - 0.2}.w_j = 0.7w_j.$$

Now,

$$w^{t'} = (0.3, 0.4, 0.225, 0.075).$$

In TOPSIS technique ,this changes in the weight affects the weighted normalized matrix, and then we have

$$w^{t'} = \left(0.3, \quad 0.4, \quad 0.225, \quad 0.075\right)$$

$$D' = \begin{bmatrix} 13 & 9 & 9 & 8 \\ 5 & 3 & 5 & 12 \\ 7 & 5 & 7 & 6 \end{bmatrix}$$

Normalized matrix, $r_{ij} = \dfrac{d_{ij}}{\sqrt{\sum_{i=1}^{m} d_{ij}^2}}$

$$R = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{bmatrix} 0.83 & 0.84 & 0.72 & 0.51 \\ 0.38 & 0.28 & 0.40 & 0.77 \\ 0.41 & 0.47 & 0.56 & 0.38 \end{bmatrix} \end{array}$$

Weighted normalized matrix is

$$v' = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{bmatrix} 0.83*0.4 & 0.84*0.2 & 0.72*0.3 & 0.51*0.1 \\ 0.38*0.4 & 0.28*0.2 & 0.40*0.3 & 0.77*0.1 \\ 0.41*0.4 & 0.47*0.2 & 0.56*0.3 & 0.38*0.1 \end{bmatrix} \end{array}$$

$$\bar{v} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{bmatrix} 0.245 & 0.336 & 0.163 & 0.038 \\ 0.114 & 0.112 & 0.090 & 0.057 \\ 0.124 & 0.186 & 0.126 & 0.029 \end{bmatrix} \end{array}$$

Since J={1,2},J'={1,4} then ,ideal and anti-ideal solutions are calculated.

$$A^+ = \{0.114, 0.336, 0.163, 0.029\},\ A^- = \{0.248, 0.112, 0.09, 0.057\}$$

$$\boxed{d_1^+ = 0.134}\ \boxed{d_2^+ = 0.237}\ \boxed{d_3^+ = 0.154}$$

$$\boxed{d_1 - = 0.236}\ \boxed{d_2^- = 0.134}\ \boxed{d_3^- = 0.151}$$

$$cl_i^{+'} = \frac{d_i^{-'}}{d_i^-{'} + d_i^{+'}}, i = 1, 2, 3,$$

$$cl_1^{+'} = \frac{d_1^{-'}}{d_1^-{'} + d_1^{+'}} = \frac{0.236}{0.236 + 0.134}\quad \boxed{cl_1^{+'} = 0.636}$$

$$\boxed{cl_2^{+'} = 0.361}\ \boxed{cl_3^{+'} = 0.495}\qquad \text{So } \boxed{A_1 > A_3 > A_2}.$$

It is obvious that, the ranking of alternatives has changed because of changing in the weight of the second attribute. we can calculate the final score of alternative by considering the changes in the weight of second attribute as,

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$$d_i'^{+} = \left\{ \sum_{j=1}^{k} ((v_{ij}' - v_j'^{+})^2)^{(1/2)} \right\}$$

$$= \left\{ \sum_{j=1 j\neq P}^{k} (v_{ij} - v_j'^{+})^2 . \gamma^2 + (v_{ip} + \Delta_p . r_{ip} - v_p^{+} - \Delta_p . r_{lp})^2 \}^{(1/2)} \right\}$$

$$d_i'^{-} = \left\{ \sum_{j=1}^{k} ((v_{ij}' - v_j'^{-})^2)^{(1/2)} \right\}$$

$$= \left\{ \sum_{j=1 j\neq P}^{k} (v_{ij} - v_j'^{-})^2 . \gamma^2 + (v_{ip} + \Delta_p . r_{ip} - v_p^{-} - \Delta_p . r_{lp})^2 \}^{(1/2)} \right\}$$

where,

$$\gamma = \frac{1 - w_p'}{1 - w_p} = \frac{1 - w_2'}{1 - w_2}$$

$$\boxed{\gamma = 0.75}.$$

With regard to the matrixes R,V in primal model,

$v_2^{+} = 0.17,\ v_2^{-} = 0.06,$

$r_{l2} = 0.84,\ r_{l2}' = 0.28$.

By replacing these values in above equation we have,

$d_l'^{+} = 0.135, d_2'^{+} = 0.237,\ d_3'^{+} = 0.154.$

$d_l'^{-} = 0.236, d_2'^{-} = 0.134,\ d_3'^{-} = 0.152.$

And from the equation $cl_i^{+'} = \dfrac{d_i^{-'}}{d_i^{-'} + d_i^{+'}}$ can be calculate $cl_i^{+'}$ as:

$cl_1^{+'} = 0.636,$

$cl_2^{+'} = 0.362,$

$cl_3^{+'} = 0.497.$

So the final rank of alternatives would be $\boxed{A_1 > A_3 > A_2}$.that is exactly the same result obtained by resolving problem.

NOTE: The ratio of new and old weight of all attributes except attribute 2will not changes that is:

$$\frac{W_i'}{W_j'} = \frac{W_i}{W_j}; i, j = 1,3,4$$

For example,for attribute 1st and4th we have,

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$$\frac{W_1'}{W_4'} = \frac{W_1}{W_4} \Rightarrow \frac{0.3}{0.075} = \frac{0.4}{0.1} = 4$$

Method-3: The application of entropy method to determine the weight of each indicators

Entropy was originally a thermodynamic concept, first introduced into information theory by Shannon. It has been widely used in the engineering, socioeconomic and other fields. According to the basic principles of information theory, information is a measure of systems ordered degree, and the entropy is a measure of systems disorder degree.

Step1: Calculate p_{ij} (the i^{th} schemes j^{th} indicators values proportion).

$p_{ij}=r_{ij}/\sum r_{ij}$, r_{ij} is the i^{th} schemes j^{th} indicators value.

Step2: Calculate the j^{th} indicators entropy value e_j, $e_j=-k\sum p_{ij}\ln p_{ij}$,

k=1/ln m, m is the number of assessment schemes.

Step3: Calculate weight w_j(j^{th} indicators weight).

$w_j=(1-e_j)/\sum(1-e_j)$, n is the number of indicators,and $0\le w_j\le 1$, $\sum w_j=1$.In entropy method, the smaller the indicators entropy value e_j is, the bigger the variaation extent of assessment value of indicators is, the more the amount of information provided, the greater the role of the indicator in the comprehensive evaluation, the higher its weight should be.

$$p_{ij} = \frac{r_{ij}}{\sum_{j=1}^{m} r_{ij}}$$

$$e_j = -k\sum_{i=1}^{m} p_{ij}\ln p_{ij}, \quad k = 1/\ln m$$

$$p_{11}\ln p_{11} = -0.3426,$$
$$p_{21}\ln p_{21} = -0.3401,$$
$$p_{31}\ln p_{31} = -0.3478,$$

$$e_1 = -1.0305 * 0.91023 = -0.9379,$$
$$e_2 = -1.0032 * 0.91023 = -0.9131,$$
$$e_3 = -1.0544 * 0.91023 = -0.9597,$$
$$e_4 = -1.0604 * 0.91023 = -0.9653,$$

$$(1-e_1) = (1-0.9379) = 1.9379,$$
$$(1-e_2) = (1-0.9131) = 1.9131,$$
$$(1-e_3) = (1-0.9597) = 1.9597,$$
$$(1-e_4) = (1-0.9653) = 1.9653.$$

$$P_{14} lnp_{14} = -0.3614,$$
$$P_{24} lnp_{24} = -0.3615,$$
$$P_{34} lnp_{34} = -0.3375,$$

$$w_1 = \frac{(1-e_1)}{\sum_{j=1}^{n}(1-e_1)},\ w_2 = \frac{(1-e_2)}{\sum_{j=1}^{n}(1-e_2)},\ w_3 = \frac{(1-e_3)}{\sum_{j=1}^{n}(1-e_3)},\ w_4 = \frac{(1-e_4)}{\sum_{j=1}^{n}(1-e_4)}$$

$$\boxed{w_1 = 0.2492}\ \boxed{w_2 = 0.2460}\ \boxed{w_3 = 0.2520}\ \boxed{w_4 = 0.2528}$$

$$w^t = \{0.2492, 0.2460, 0.2520, 0.2528\}$$

weighted normalized matrix is:

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$$R = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{bmatrix} 0.83 & 0.84 & 0.72 & 0.51 \\ 0.38 & 0.28 & 0.40 & 0.77 \\ 0.41 & 0.47 & 0.56 & 0.38 \end{bmatrix} \end{array}$$

$$v' = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{bmatrix} 0.83*2492 & 0.84*2460 & 0.72*2520 & 0.51*2528 \\ 0.38*2492 & 0.28*2460 & 0.40*2520 & 0.77*2528 \\ 0.41*2492 & 0.47*2460 & 0.56*2520 & 0.38*2528 \end{bmatrix} \end{array}$$

$$v' = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{bmatrix} 0.2068 & 0.2066 & 0.1814 & 0.1289 \\ 0.0947 & 0.0689 & 0.1008 & 0.1946 \\ 0.1022 & 0.1156 & 0.1411 & 0.0961 \end{bmatrix} \end{array}$$

ii)Determine ideal solution A^+:

since J={1,2} ,J'={1,4} then ideal and anti-ideal solution would be,

$$A^+ = \{0.0947, 0.2066, 0.1814, 0.0961\}$$

$$v' = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{bmatrix} 0.2068 & 0.2066 & 0.1814 & 0.1289 \\ 0.0947\downarrow & 0.0689\uparrow & 0.1008\uparrow & 0.1946\downarrow \\ 0.1022 & 0.1156 & 0.1411 & 0.0961 \end{bmatrix} \end{array}$$

iii)Negative ideal solution A^-:

$$A^- = \{0.2068, 0.0689, 0.1008, 0.1946\}$$

$$v' = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{bmatrix} 0.2068 & 0.2066 & 0.1814 & 0.1289 \\ 0.0947\uparrow & 0.0689\downarrow & 0.1008\downarrow & 0.1946\uparrow \\ 0.1022 & 0.1156 & 0.1411 & 0.0961 \end{bmatrix} \end{array}$$

By using the Euclidean norm, distance of alternatives from ideal and anti-ideal solutions are;

$$d_i^+ = \{\sum (v_{ij} - v_j^+)^2\}^{(1/2)},$$

$$d_i^- = \{\sum (v_{ij} - v_j^-)^2\}^{(1/2)}.$$

$\boxed{d_1^+ = 0.1166}$ $\boxed{d_2^+ = 0.1876}$ $\boxed{d_3^+ = 0.0995}$

$\boxed{d_1^- = 0.1726}$ $\boxed{d_2^- = 0.1122}$ $\boxed{d_3^- = 0.1562}$

Then the final score of alternatives are calculated by,

$$cl_i^+ = \frac{d_i^-}{d_i^- + d_i^+}, i = 1, 2, 3$$

$$cl_1^+ = \frac{d_1^-}{d_1^- + d_1^+} = \frac{0.1726}{0.1726 + 0.1166}$$

$$\boxed{cl_1^+ = 0.5968}$$

$$\boxed{cl_2^+ = 0.3742}$$

$$\boxed{cl_3^+ = 0.6109}$$

So $\boxed{A_3 > A_1 > A_2}$

It is obvious that the ranking of alternatives has changed because of changing in the weight of second attributes.

Table-1: Comparison of the three methods

METHOD	**RANKING OF ALTERNATIVES**
Method-1: Using TOPSIS method	$A_3 > A_2 > A_1$
Method-2: Using TOPSIS method with Sensitivity analysis	$A_1 > A_3 > A_2$
Method-3: Using TOPSIS method with Entropy	$A_3 > A_1 > A_2$

V. CONCLUSION

The proposed research work has concentrated on issues and complexities in applying TOPSIS method to real world problems like supplier selection problems in supply chain management. The general TOPSIS method, Sensitivity analysis for TOPSIS method was proposed and new algorithm was proposed for Multiple Attribute Decision Making efficiently. The procedure for a general TOPSIS method is discussed. A case study with the theory of selecting the best supplier in a supply chain management is analyzed with the help of the proposed algorithm of TOPSIS method extended with a sensitivity analysis with changes taking place in weighting vector is presented. A numerical illustration is presented utilizing the TOPSIS method for supplier selection problem together with entropy method.

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