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Bi-Elliptic Hohmann Transfer and One Tangent Burn Transfer Calculations Using the Monte Carlo Simulation

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Abstract: **One of the objectives in the orbit transfer problem is to achieve the optimal time of flight and the fuel consumption for the orbital transfer maneuver between two orbits. The transfer of satellites in too high orbits as geosynchronous one (geostationary), usually is achieved firstly by launching the satellite in Low Earth Orbit (LEO) (Parking orbit), then in elliptical transfer orbit and finally to the final orbit (Working orbit).**

In this paper, the Monte Carlo Simulation will be used to determine the optimum three tangent impulses maneuver (Bi-Elliptic Hohmann transfer) and determine the optimum One Tangent Burn transfer. From respective simulation, determine the optimum altitude of the transfer tangent point for Bi-Elliptic Hohmann transfer and determine the optimum angle true anomaly (υtrans) of the One Tangent Burn transfer to create minimum change of velocity and optimum time of flight for transfer and with minimum fuel consumption of this transfer.

Keywords: **Bi-Elliptic Hohmann Transfer; Coplanar Impulsive Maneuver; Monte Carlo Simulation; One Tangent Burn Transfer; Satellite Orbit.**

1. INTRODUCTION

R. H. Goddard (1919) was one of the first researchers on the problem of optimal transfers of a spacecraft between two points who suggested optimal approximate solutions for the problem of sending a rocket to high altitudes with minimum fuel consumption [1].After that, there is the very important work done by Hohmann (1925) who solved the problem of minimum ΔV transfers between two circular coplanar orbits. His results are largely utilized nowadays as a first approximation of more complex models and it was considered the final solution of this problem until (1959) [1].

The Hohmann transfer would be generalized to the elliptic case (transfer between two coaxial elliptic orbits) by Marchal (1965) [6]. The Hohmann transfer is an elliptical orbit tangent to both circles. The perigee and apogee of the transfer ellipse are the radii of the inner and outer circles [1].Smith (1959) showed results for some other special cases, like coaxial and quasi-coaxial elliptic orbits, circular-elliptic orbits, two quasi-circular orbits [3].

To transfer from one orbit to another the velocity changes of the spacecraft are assumed to be done using propulsive systems, which occur instantaneously. Although it will take some time for the spacecraft to accelerate to the velocity of the new orbit, this assumption is reasonable when the burn time of the rocket is much smaller than the period of the orbit [4]. The Hohmann transfer is well known for the minimum of propellant mass used for satellite transfer into high orbits.

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2. DEFINITION OF THE PROBLEM

The problem to be studied is that of optimal transfer of a satellite between a pair of coplanar elliptical orbits, employing three impulsive thrusts or one tangent burn. The objective of this problem is to modify the optimum orbit transfer of a given spacecraft. The problem is using Monte Carlo Simulation to the optimum altitude of the transfer tangent point for Bi-Elliptic Hohmann transfer and determine the optimum angle true anomaly (υ_{trans}) of the One Tangent Burn transfer to create minimum change of velocity and optimum time of flight for transfer and with minimum fuel consumption of this transfer.

3. ELLIPTIC ORBIT

The path of the satellite's motion is an orbit. Generally, the orbits of communication satellites are ellipses laid on the orbital plane defined by space orbital parameters. These parameters (Kepler elements) determine the position of the orbital plane in space, the location of the orbit within orbital plane and finally the position of the satellite in the appropriate orbit [2], [8]. The elliptic orbit is determined by the semi-major axis which defines the size of an orbit, and the eccentricity which defines the orbit's shape. Orbits with no eccentricity are known as circular orbits. The elliptic orbit shaped as an ellipse, with a maximum extension from the Earth center at the apogee (r_a) and the minimum at the perigee (r_p) is presented in Figure (1).

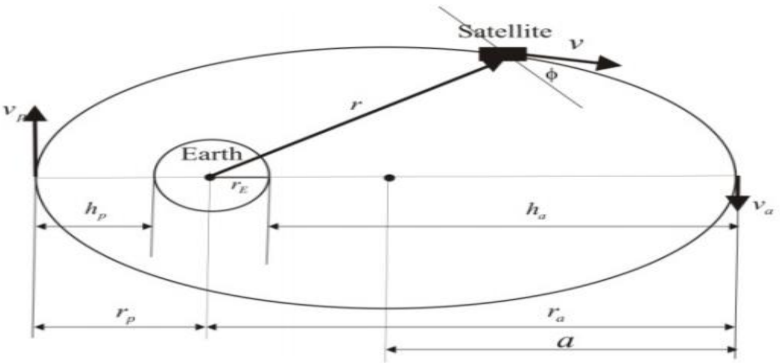

Figure (1): Major parameters of an elliptic orbit.

The orbit's eccentricity is defined as the ratio of difference to sum of apogee (ra) and perigee (rp) radii as, [2] - [9].

$$e = \frac{r_a - r_p}{r_a + r_p} \qquad (1)$$

Applying geometrical features of ellipse yield out the relations between semi major axis, apogee and perigee:

$$r_p = a(1 - e) \qquad (2)$$

$$r_a = a(1 + e) \qquad (3)$$

$$2a = r_a + r_p \qquad (4)$$

Both, r_p and r_a are considered from the Earth's center. Earth's radius is

$r_E = 6378$ km . Then, the altitudes (highs) of perigee and apogee are:

$$H_p = r_p - r_E \qquad (5)$$

$$H_a = r_a - r_E \qquad (6)$$

Different methods are applied for satellite injection missions. Goal of these methods is to manage and control the satellite to safely reach the low Earth orbit, and then to the transfer elliptical orbit and finally the final orbit [4], [5]. The Hohmann transfer is considered as the most convenient. The specific orbit implementation depends on satellite's injection velocity. The orbit implementation process on the best way is described in terms of the cosmic velocities. Based on Kepler's laws, considering an elliptic orbit, the satellite's velocity at the perigee and apogee point.

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4. COPLANAR IMPULSIVE MANEUVERS

As the name implies, coplanar maneuvers don't change the orbital plane, so the initial and final orbits lie in the same plane. These maneuvers can change the orbit's size and shape (semi-major axis and eccentricity) and the location of the line of apsides (argument of perigee). Coplanar bums are either tangential or non-tangential. All of the historical work in coplanar transfers was restricted to circular orbits for which the velocity vector is always tangential to the orbit.

Impulsive maneuvers are those in which brief firings of onboard rocket motors change the magnitude and direction of the velocity vector instantaneously. During an impulsive maneuver, the position of the spacecraft is considered to be fixed; only the velocity changes.

The impulsive maneuver is an idealization by means of which we can avoid having to solve the equations of motion with the rocket thrust included. The idealization is satisfactory for those case s in which the position of the spacecraft changes only slightly during the time that the maneuvering rockets fire. This is true for high thrust rockets with burn times that are short compared with the coasting time of the vehicle.

4.1 Three impulses tangent maneuver (Bi-Elliptic Hohmann transfer):

To transfer from elliptical orbit (initial) to elliptical orbit (final) by Bi-Elliptic Hohmann transfer (three tangent impulses), we have two cases:

a) The first pulse in perigee point of initial orbit (point A) and the second pulse in apogee point of transfer orbit (point C) and third pulse in perigee point of final orbit (point B), Figure (2). [7].

b) The first pulse in apogee point of initial orbit (point A) and the second pulse in apogee point of transfer orbit (point C) and third pulse in apogee point of final orbit (point B), Figure (3). [7].

For this method we know parameters of initial orbit and final orbit but we want to determine the altitude of the second point (C) which is applying the second tangent pulse on it and this is the main target of the paper to evaluate the optimum altitude for minimum total velocity for change orbits by using Monte Carlo Simulation.

- ***Cases (a):***

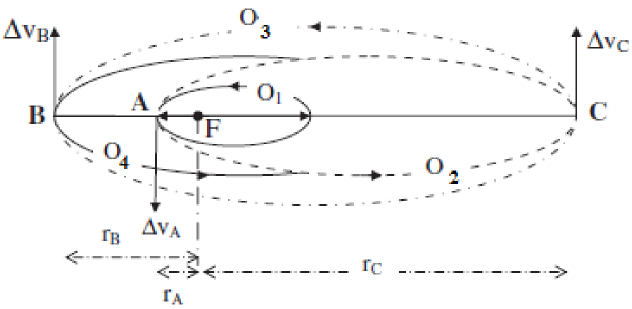

Figure (2): Bi-Elliptic Hohmann transfer from perigee point of initial orbit

According to the figure (2) we have

- Delta-V for transfer:

Using Monte Carlo Simulation to determine altitude of point (C) r_{a2} (apogee radius of the orbit 2)

- At point A

$$v_A)_1 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a1}r_{p1}}{r_{a1}+r_{p1}}}}{r_{p1}} \text{(km/sec)} \quad \text{For orbit (1)} \qquad (7)$$

$$v_A)_2 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a2}r_{p1}}{r_{a2}+r_{p1}}}}{r_{p1}} \text{(km/sec)} \quad \text{For orbit (2)} \qquad (8)$$

$$\therefore \Delta V_A = |v_A)_2 - v_A)_1| \text{ (km/sec)} \qquad (9)$$

- At point C

$$v_C)_2 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a2}r_{p1}}{r_{a2}+r_{p1}}}}{r_{a2}} \text{(km/sec)} \quad \text{For orbit (2)} \qquad (10)$$

$$v_C)_3 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a2}r_{p4}}{r_{a2}+r_{p4}}}}{r_{a2}} \text{(km/sec)} \quad \text{For orbit (3)} \qquad (11)$$

$$\therefore \Delta V_C = |v_C)_3 - v_C)_2| \text{ (km/sec)} \qquad (12)$$

- At point B

$$v_B)_3 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a2}r_{p4}}{r_{a2}+r_{p4}}}}{r_{p4}} \text{(km/sec)} \quad \text{For orbit (3)} \qquad (13)$$

$$v_B)_4 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a4}r_{p4}}{r_{a4}+r_{p4}}}}{r_{p4}} \text{(km/sec)} \quad \text{For orbit (4)} \qquad (14)$$

$$\therefore \Delta V_B = |v_B)_3 - v_B)_4| \text{ (km/sec)} \qquad (15)$$

The total delta-V requirement for this Bi-elliptic Hohmann transfer is

$$\Delta V_{total} = |\Delta V_A| + |\Delta V_C| + |\Delta V_B| \text{ (km/sec)} \qquad (16)$$

- <u>The time of flight</u>

The semi-major axis of the transfer ellipse is

$$a_{t1} = \frac{1}{2}\left(r_{p1} + r_{a2}\right) \text{ (km)} \qquad (17)$$

$$a_{t2} = \frac{1}{2}\left(r_{p4} + r_{a2}\right) \text{ (km)} \qquad (18)$$

$$T = \frac{2\pi}{\sqrt{\mu}}\, a_{t1}^{\frac{3}{2}} + \frac{2\pi}{\sqrt{\mu}}\, a_{t2}^{\frac{3}{2}} \text{ (sec)} \qquad (19)$$

The time of flight for this Bi-elliptic Hohmann transfer is

$$\therefore \Delta t_{bi-elliptical} = \frac{1}{2} T \text{ (sec)} \qquad (20)$$

- ***<u>Cases (b)</u>***

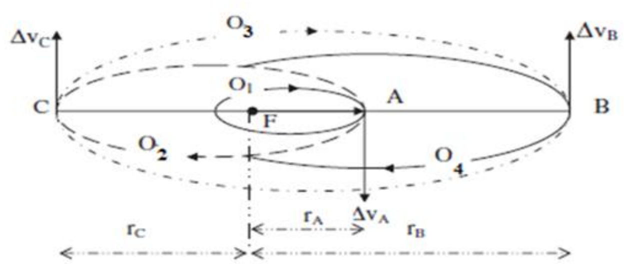

Figure (3): Bi-Elliptic Hohmann transfer from perigee point of initial orbit

According to the figure (3) we have

- Delta-V for transfer

Using Monte Carlo Simulation to determine altitude of point (C) r_{a2} (apogee radius of the orbit 2)

- At point A

$$v_A)_1 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a1}r_{p1}}{r_{a1}+r_{p1}}}}{r_{a1}} \text{ (km/sec)} \quad \text{For orbit (1)} \qquad (21)$$

$$v_A)_2 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a2}r_{a1}}{r_{a2}+r_{a1}}}}{r_{a1}} \text{ (km/sec)} \quad \text{For orbit (2)} \qquad (22)$$

$$\therefore \Delta V_A = |v_A)_2 - v_A)_1| \text{ (km/sec)} \qquad (23)$$

- At point C

$$v_C)_2 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a2}r_{a1}}{r_{a2}+r_{a1}}}}{r_{a2}} \text{ (km/sec)} \quad \text{For orbit (2)} \qquad (24)$$

$$v_C)_3 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a2}r_{a4}}{r_{a2}+r_{a4}}}}{r_{a2}} \text{ (km/sec)} \quad \text{For orbit (3)} \qquad (25)$$

$$\therefore \Delta V_C = |v_C)_3 - v_C)_2| \text{ (km/sec)} \qquad (26)$$

- At point B

$$v_B)_3 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a2}r_{a4}}{r_{a2}+r_{a4}}}}{r_{a4}} \text{ (km/sec)} \quad \text{For orbit (3)} \qquad (27)$$

$$v_B)_4 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a4}r_{p4}}{r_{a4}+r_{p4}}}}{r_{a4}} \text{ (km/sec)} \quad \text{For orbit (4)} \qquad (28)$$

$$\therefore \Delta V_B = |v_B)_3 - v_B)_4| \text{ (km/sec)} \qquad (29)$$

The total delta-V requirement for this Bi-elliptic Hohmann transfer is

$$\Delta V_{total} = |\Delta V_A| + |\Delta V_C| + |\Delta V_B| \text{ (km/sec)} \qquad (30)$$

- The time of flight

The semi-major axis of the transfer ellipse is

$$a_{t1} = \frac{1}{2}(r_{a1} + r_{a2}) \text{ (km)} \qquad (31)$$

$$a_{t2} = \frac{1}{2}(r_{a2} + r_{a4}) \text{ (km)} \qquad (32)$$

$$T = \frac{2\pi}{\sqrt{\mu}} a_{t1}^{\frac{3}{2}} + \frac{2\pi}{\sqrt{\mu}} a_{t2}^{\frac{3}{2}} \text{ (sec)} \qquad (33)$$

The time of flight for this Bi-elliptic Hohmann transfer is

$$\therefore \Delta t_{bi-elliptical} = \frac{1}{2} T \text{ (sec)} \qquad (34)$$

4.2 One Tangent Burn transfer:

The major drawback to the Hohmann transfer is the long flight time. The bi-elliptic takes even longer. To reduce time of flight, we must select a trajectory using a shorter path with higher velocities. The fastest possible path would involve a Δv; approaching infinity, but that's not practical. Thus, we must select a trajectory that reduces time of flight at the expense of an acceptable increase in Δv. One solution is the one-tangent bum. As the name implies, a one-tangent burn has one tangential bum and one non-tangential bum.

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This method reduces the transfer time of the Hohmann techniques but increases Δv requirements [11]. Note that we must know the transfer orbit's true anomaly (semi-major axis, or eccentricity) to locate the non-tangential burn. Figure (4) shows the situation of transfer from elliptical orbit (initial) to elliptical orbit (final) by using one tangent burn and Monte Carlo simulation.

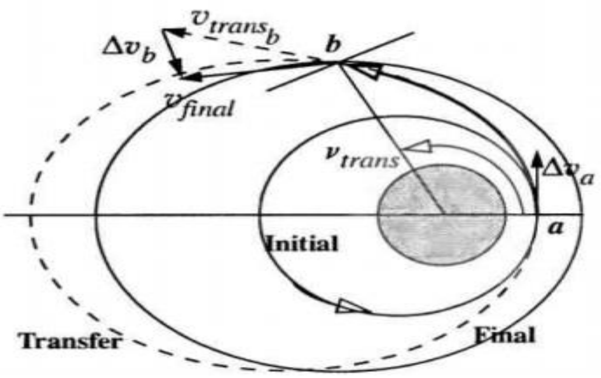

Fig. (4): One tangent burn transfer

According to the figure (4) we have

- Input
  - Orbit (1) initial

r_{p1} (perigee radius of orbit (1))

r_{a1} (apogee radius of orbit (1))

- Orbit (4) final

r_{p2} (perigee radius of orbit (2))

r_{a2} (apogee radius of orbit (2))

μ (Gravitational parameter) = 3.986012 x 105 km3/sec2

- Steps for calculation [7].

Using Monte Carlo Simulation to determine $v_{trans\,b}$

Not that $v_{trans\,b} \neq \pm \cos^{-1}(R)$

$$R = \frac{r_{initial}}{r_{final}} \tag{35}$$

$$R = \frac{r_{p1}}{r_{p2}} \tag{36}$$

Eccentricity and semi major axis of transfer orbit are

$$\therefore e_{trans} = \frac{R-1}{\cos v_{trans} - R} \tag{37}$$

$$and\ a_{trans} = \frac{r_{p1}}{1-e_{trans}} \tag{38}$$

- Delta-V for transfer

Compute the velocities

- At point *a*

$$V_a)_1 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a1}r_{p1}}{r_{a1}+r_{p1}}}}{r_{p1}} (\text{km/sec}) \quad \text{For orbit (1)} \tag{39}$$

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$V_{trans\,a})_3 = \sqrt{\frac{(2\mu)}{r_{p1}} - \frac{\mu}{a_{trans}}}$ (km/sec) For orbit (3) (40)

$\therefore \Delta V_a = |V_{trans\,a})_3 - V_{Aa})_1|$ (km/sec) (41)

- At point b

$a_2 = \frac{r_{a2}+r_{p2}}{2}$ (42)

$r_b = \frac{a_{trans}(1-e_{trans}^2)}{1+e_{trans}\cos(v_{trans})}$ (43)

$V_b)_2 = \sqrt{\left(\frac{(2\mu)}{r_b} - \frac{\mu}{a_2}\right)}$ (km/sec) For orbit (2) (44)

$V_{trans\,b})_3 = \sqrt{\left(\frac{(2\mu)}{r_b} - \frac{\mu}{a_{trans}}\right)}$ (km/sec) For orbit (3) (45)

$\tan\varphi_{trans\,b} = \frac{e_{trans} * \sin(v_{trans\,b})}{1+(e_{trans} * \cos(v_{trans\,b}))}$ (46)

$\Delta V_b = \sqrt{[V_b)_2]^2 + [V_{trans\,b})_3]^2 - [2 * V_b)_2 * V_{trans\,b})_3 * \cos(\varphi_{trans\,b})]}$ (47)

The total delta-V requirement for this transfer is

$\Delta V_{total} = |\Delta V_a| + |\Delta V_b|$(km/sec) (48)

- The time of flight

$\cos(E) = \frac{e_{trans}+\cos(v_{trans\,b})}{1+(e_{trans} * \cos(v_{trans\,b}))}$ (49)

$t_{trans} = \sqrt{\frac{a_{trans}^3}{\mu}}\{(2k\pi) + (E - (e_{trans} * \sin(E))) - (E_0 - (e_{trans} * \sin(E_0)))\}$ (50)

Because this transfer starts at periapsis, so E0=0o. The transfer doesn't pass perigee, so K must equal zero

$\therefore t_{trans} = \sqrt{\frac{a_{trans}^3}{\mu}}\{(E - (e_{trans} * \sin(E)))\}$ (51)

5. NUMERICAL RESULTS

The transfer is initiated by firing the spacecraft engine at low Earth orbit in order to accelerate it so that it will follow the elliptical orbit; this adds energy to the spacecraft's orbit. When the spacecraft has reached transfer orbit, its orbital speed (and hence its orbital energy) must be increased again in order to change the elliptic orbit to the final orbit. In Bi-Elliptic Hohmann transfer we will this transfer by three tangent impulses but in one tangent burn transfer we will this transfer by two impulses the first impulse is tangent and the second impulse is non-tangent:

According Valado D. A., 2001 "Fundamentals of Astrodynamics and Application" [9], solved example at chapter 5 point 5.3 page 292. We compared between this data and Monte Carlo Simulation to solve Bi-elliptic Hohmann transfer in table (1) and solve One-Tangent Burn transfer in table (2)

4.3 Bi-Elliptic Hohmann Transfer by Using Monte Carlo Simulation:

Table (1): Variation between the default calculations and Monte Carlo Simulation for Bi-Elliptic Hohmann transfers

	Initial Alt. (km)	Final Alt. (km)	Transfer Alt. (km)	ΔV (km/sec)	T_{trans} (hour)
Transfer to Geosynchronous					
By default	191.34411	35781.35	47836	4.076	21.944
Monte Carlo	191.34411	35781.35	35791.35	3.93543	17.22561
Transfer to the moon					
By default	191.34411	376310	503873	3.904	593.919

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Monte Carlo	191.34411	376310	605923	3.86961	722.94293
Transfer to high altitude					
By default	622	98622	203622	4.0285	135.79677
Monte Carlo	622	98622	245622	4.01931	168.65486

4.4 One-Tangent Burn Transfer by Using Monte Carlo Simulation:

Table (2): Variation between the default calculations and Monte Carlo Simulation for One-tangent burn transfers

	Initial Alt. (km)	**Final Alt. (km)**	**True anomaly transfer**	**ΔV (km/sec)**	**T_{trans} (hour)**
Transfer to Geosynchronous					
By default	191.34411	35781.35	160˚	4.699	3.457
Monte Carlo	191.34411	35781.35	178.9575˚	3.93763	5.12620
Transfer to the moon					
By default	191.34411	376310	175˚	4.099	83.061
Monte Carlo	191.34411	376310	178.9575	3.97236	109.13704
Transfer to high altitude					
By default	622	98622	160˚	5.05983	9.79605
Monte Carlo	622	98622	178.9575˚	4.04973	17.57563

4.5 Comparison of Coplanar Orbital Transfers (Bi-Elliptic Hohmann Transfer and One-Tangent Burn Transfer) by Using Monte Carlo Simulation:

The table (3) presents results of the trade-off between change in velocity and time of flight for different types of orbital changes (One- tangent burn transfer using Monte Carlo Simulation and Bi-Elliptic Hohmann transfer using Monte Carlo Simulation). Recognize that the initial and final orbits are circular, whereas the transfer orbits are elliptical, with perigee and apogee values as shown.

Table (3): Comparison of Coplanar Orbital Transfers using Monte Carlo

	Initial Alt. (km)	**Final Alt. (km)**	**Transfer orbit (degree),(km)**	**ΔV (km/sec)**	**T_{trans} (hour)**
Transfer to Geosynchronous					
One-tangent	191.34411	35781.35	υ_{trans} =160˚	4.699	3.457
Bi-Elliptic	191.34411	35781.35	Alt. = 47836	4.076	21.944
Transfer to the moon					
One-tangent	191.34411	376310	υ_{trans} =175˚	4.099	83.061
Bi-Elliptic	191.34411	376310	Alt. = 503873	3.904	593.919
Using Monte Carlo Simulation					
Transfer to Geosynchronous					
One tangent	191.34411	35781.35	υ_{trans} = 178.9575	3.938	5.126
Bi-Elliptic	191.34411	35781.35	Alt. = 35791.35	3.93543	17.226
Transfer to the moon					
One-tangent	191.34411	376310	υ_{trans} = 178.96	3.972	109.137
Bi-Elliptic	191.34411	376310	Alt. = 605923	3.870	722.943

6. CONCLUSION

One tangent Burn: as the name implies, a one-tangent burn has one tangential burn and one non-tangential burn. This method reduces the transfer time of the Hohmann techniques but increases ΔV requirements.

According the calculations in table (1), Bi-Elliptic Hohmann Transfer by Using Monte Carlo Simulation we found that in the transfer to Geosynchronous the ΔV and time of flight for transfer decreased and in transfer to the moon the ΔV decreased but the time of flight for transfer increased and in the last case also the same which the ΔV decreased but the time of flight for transfer increased.

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According the calculations in table (2), One-Tangent Burn Transfer by Using Monte Carlo Simulation we found that the ΔV for transfer decreased but the time of flight for transfer increased.

According the calculations in table (3), the transformation of spacecraft to Geosynchronous the three impulse (bi-elliptic transfer) is more economical compare with one tangent burn. Consequently less fuel is needed to be carried out on the spacecraft in case that the spacecraft initially is injected on the higher altitude (transfer to Geosynchronous) but the time of flight is the middle of the range of times.

The transformation of spacecraft to the moon the Bi-elliptic Hohmann transfer (three impulses) is optimum compare with one tangent burn transfer but the time of flight for this transfer is larger than one tangent burn. One tangent burn is the largest ΔV but the time of flight is the least one. One tangent burn is the fastest method which take less time of flight compare with and Bi-elliptic Hohmann transfer.

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