Comparison of ARIMA & Circular Model in Forecasting Sri Lankan Share Market Returns

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Abstract: Share market has a prominent place in the economy of a country. In Sri Lankan context, share trading contributes one third of the Gross Domestic Product. Share market investments are considered as high return investments, but in cooperated with high risk. As such, scientific forecasting takes the place of a light house in share market investments. Scientific forecasting techniques used in share markets can be subdivided into two parts; statistical techniques and soft computing techniques. Statistical techniques are categorized in to univariate techniques and multivariate techniques. This study was focused on univariate statistical techniques. The study compares the forecasting ability of Auto Regressive Integrated Moving Average Model and Circular Model in forecasting Sri Lankan share market returns.

Keywords: Auto Regressive Integrated Moving Average Model, Circular Model.

I. INTRODUCTION

Firms have two types of assets; real assets and financial assets. Real assets are physical assets such as plant, machinery, building etc. and technological know-how, patents, copyrights etc. Financial assets or securities are financial papers or instruments such as shares and bonds. Firms issue securities to investors in the primary capital markets. The securities issued by firms are traded in the secondary capital markets, referred to as stock exchanges.

Return of a single asset or share is defined as the sum of Dividend yield and Capital gain (value of sold shares). Dividend yield totally depends on the dividend decision of firms. As such major concern of the investors is capital gain yield. This would have been the stimulation for research in stock market forecasting. On the other hand, stock market investments make high returns at high risk. Therefore forecasting is essential for the investors to optimize their wealth.

Statistical forecasting plays the major role in share market. It comprises univariate techniques and multivariate techniques. A univariate statistical model is an equation or set of equations explaining the behavior of a single random variable over time while the multivariate statistical models explain the joint behavior of two or more random variables. The univariate statistical modeling procedure is based on the past internal patterns in data to forecast the future and no external variables are required in forecasting. The basic concept of these methods is that future values of a series are a function of past values. Auto Regressive Integrated Moving Average (ARIMA) models are a well known group of univariate models.

PROBLEM STATEMENT:

ARIMA models can be applied only for stationary series. In other words, this technique may suitable for series which have wave like patterns. According to literature, ARIMA models have a significant position in share market forecasting. They are known to be efficient and robust in forecasting share prices or share returns. Rosangela, Ivette, Lilian, &
Rodrigo (2010); Ayodele, Aderemi & Charles (2014-a); Prapanna, Labani & Saptarsi (2014); Emenike (2014); Konarasinghe, Abeynayake & Gunaratne (2015-a) and many others have given evidences for the success of ARIMA in forecasting share prices or returns. Ayodele, Aderemi & Charles (2014-b) also agreed with the efficiency of ARIMA in forecasting share prices, but pointed out a common weakness in ARIMA forecasts. According to Ayodele et al. (2015), ARIMA forecasts do not follow the patterns of actual share prices.

Konarasinghe, Abeynayake & Gunaratne (2016) have suggested a model, named “Circular Model”, in forecasting returns. It also comes under univariate forecasting techniques, suitable for wave like patterns. This study was focused on comparison of ARIMA and Circular model in Sri Lankan context.

II. METHODOLOGY

Listed companies of Colombo Stock Exchange (CSE) in year 2014 were the population of the study. The population consists of 294 companies in 20 business sectors. Daily closing share prices of individual companies from year 1991 to year 2014 were obtained from CSE data library. A random sample of twenty companies was selected for analysis.

Monthly average share prices for individual companies were calculated by daily closing share prices. One standard formula for calculating share return at time t is;

\[ R_t = \left( \frac{P_t - P_{t-1}}{P_{t-1}} \right) \times 100 \]  

(1)

Where; \( P_t \) is the share price at time \( t \) (Pande, 2005). Outlier adjusted data were tested on ARIMA model and Circular Model.

Statistical Methods Used in the Study:

ARIMA model and Circular model are the tested statistical models. Goodness of fit tests and measurements of errors are the techniques used in model validation.

ARIMA model:

ARIMA models come under the General Linear Process. A General Linear Process is a stationary stochastic process \( \{ Y_t \} \) which can be represented as weighted linear combination of the present and past terms of a white noise. These include Auto Regressive (AR), Moving Average (MA), Auto Regressive Moving Average (ARMA) and Auto Regressive Integrated Moving Average (ARIMA).

\( AR_p \) Model:

Auto Regressive Process \( \{ Y_t \} \) of order \( p \) has the model; \( Y_t = c + \sum_{i=1}^{p} \phi_i Y_{t-i} + \epsilon_t \)  

(2)

Where \( Y_{t-i} \) are past observations of random variable \( Y_t \).

\( MA_q \) Model:

Moving Average Process \( \{ Y_t \} \) of order \( q \) has the model; \( Y_t = \mu + \epsilon_t + \sum_{i=1}^{q} \theta_i \epsilon_{t-i} \)  

(3)

Where \( \epsilon_{t-i} \) are past errors of random variable \( Y_t \).

\( ARMA_{p,q} \) Model:

A model containing both AR and MA parts is known as a mixed model or the Auto Regressive Moving Average (ARMA) model. ARMA (\( p,q \)) model is:

\[ Y_t = c + \epsilon_t + \sum_{i=1}^{q} \theta_i \epsilon_{t-i} + \sum_{i=1}^{p} \phi_i Y_{t-i} \]

(4)
ARIMA p, d, q Model:
ARMA model is not valid if the series is not stationary. Therefore, differences are taken in order to achieve stationary and order of difference is given by d. Then the model is called Auto Regressive Integrated Moving Average model, given by the equation:

\[ \phi_p (B) \Delta^d Y_t = \theta_q (B) \epsilon_t \]  

(5)

Where B is the back shift operator.

Stationary Stochastic Process:
A stochastic process with a discrete time parameter is said to stationary (or stationary in the strict sense) if the distribution of \( y_{t_1}, y_{t_2}, \ldots, y_{t_n} \) is the same as the distribution of \( y_{t_1+t}, y_{t_2+t}, \ldots, y_{t_n+t} \) for every finite set of integers \( \{t_1, t_2, \ldots, t_n\} \) and for every integer t (Anderson, 1971). A stochastic process is said to be stationary, if its mean and variance are constants over time and the value of the covariance between the two periods depends only on the distance (gap or lag) between the two time periods and not the actual time at which the covariance is computed.

CIRCULAR MODEL:
Circular Model (CM) combines Fourier transformation and multiple regression analysis.

Fourier Transformation:
Fourier transformation is a linear transformation. It has two versions; discrete transformation and continuous transformation. Discrete version of Fourier transformation is:

\[ f(x) = \sum_{n=-\infty}^{\infty} a_n e^{-k\theta} \]  

(6)

According to De Moivre's theorem; \( e^{-k\theta} = \cos k\theta + i \sin k\theta \)  

(7)

Where, \( i \) is a complex number. Therefore \( f(x) \) is written as:

\[ f_x = \sum_{k=1}^{n} a_k \cos k\theta + b_k \sin k\theta \]  

(8)

Where \( a_k \) and \( b_k \) are amplitudes; k is the harmonic of oscillation.

Fourier transformation is in cooperated to a uniform circular motion of a particle in a horizontal circle and basic trigonometric ratios. A particle \( P \) which is moving in a horizontal circle of centre \( O \) and radius \( a \) is given in Figure 1. \( \omega \) is the angular speed of the particle at time \( t \). In circular motion, a time taken for one complete circle is known as the period of oscillation. In other words, period of oscillation is equal to the time between two peaks or troughs of sine or cosine function.

Figure 1: Motion of a particle in a horizontal circle
If a time series follows a wave with \( f \) peaks in \( N \) observations, its period of oscillation can be given as:

\[
T = \frac{\text{total number of periods}}{\text{total number of peaks}} = \frac{N}{f} \tag{9}
\]

Then \( \omega = 2\pi \frac{f}{N} \) \tag{10}

Hence the return at time \( t \) \((R_t)\) was modeled as:

\[
R_t = \sum_{k=1}^{n} (a_k \sin k\omega t + b_k \cos k\omega t) + \varepsilon_t \tag{11}
\]

Number of observations per season (year) is \( n=12 \), hence \( k=6 \).

### III. FINDINGS

ARIMA model and Circular model were tested on random sample of twenty companies of CSE. Goodness of fit tests and Root Mean Square Error (RMSE) were used in model validation.

#### TEST ARIMA MODELS ON RETURNS:

Stationary of the series were tested by: ACFs, PACFs and Augmented Dickey Fuller Test (ADFT). ADFT was conducted in MATLAB using the syntax; \( h=\text{adftest}(Y) \). The result \( h=0 \), fails to rejects the null hypothesis of a unit root against the auto regressive alternative.

For example; ACF and PACF of returns of company HUNA had no significant spikes. ADFT result \( (h=1) \) rejects the unit root of the series. Hence it was concluded that the returns of HUNA is a stationary type. Then several ARIMA models were tested on it, results are given in Table 1. Histogram of residuals, Normal Probability plot of residuals and Anderson Darling test were used to test the Normality of residuals. Residual Vs Fits, ACF and PACF of residuals and LBQ statistics were used to test the independence of residuals.

<table>
<thead>
<tr>
<th>Model</th>
<th>P-value of the model</th>
<th>Remarks of the Model</th>
<th>Remarks of Residuals</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARIMA (0, 0, 1)</td>
<td>0.535</td>
<td>Not significant</td>
<td></td>
</tr>
<tr>
<td>ARIMA (1, 0, 0)</td>
<td>0.554</td>
<td>Not significant</td>
<td></td>
</tr>
<tr>
<td>ARIMA (0, 1, 1)</td>
<td>0.000</td>
<td>Significant, MSE=51.84</td>
<td>Normal, uncorrelated</td>
</tr>
<tr>
<td>ARIMA (1, 1, 0)</td>
<td>0.000</td>
<td>Significant, MSE=78.48</td>
<td>Normal, uncorrelated</td>
</tr>
<tr>
<td>ARIMA (1, 1, 1)</td>
<td>AR(1)-0.533</td>
<td>Not significant</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MA(1)-0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ARIMA (0, 1, 2)</td>
<td>MA(1)-0.000</td>
<td>Not significant</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MA(2)-0.059</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

ARIMA \((0, 1, 1)\) is the best model. Residuals of the model were normally distributed and independent. RMSE \((7.2)\) in model fitting and RMSE \((3.6)\) forecasting were satisfactorily small. Hence; ARIMA \((0, 1, 1)\) is a suitable model for forecasting returns of the company.

#### TEST CIRCULAR MODEL (CM) ON RETURNS:

Discrete version of the Fourier transformation, given in formula (8) is a static model. It has been applied to explain the regular waves in Physics. It is modified to describe the wave patterns associated with randomness as shown in formula (11); \( R_t = \sum_{k=1}^{n} (a_k \sin k\omega t + b_k \cos k\omega t) + \varepsilon_t \)
Amplitudes of the series, $a_k$ and $b_k$ are experimentally calculated in Physics. As it is not possible in applications, this study employs multiple regression technique for the purpose; regressing $R_t$ on $sink\omega t$ and $cosk\omega t$ for $k$ is from 1 to 6. For example, Circular model is tested on returns of company HUNA as follows;

Angular speed ($\omega$) for company HUNA is calculated by;

\[
\omega = \frac{2\pi f}{N},
\]

where $f$ is the number of peaks and $N$ is the number of observations in the series. For HUNA, $f=20$ and $N=80$ for model fitting, hence $\omega=1.5708$. Then twelve trigonometric series; $sink\omega t$ and $cos k\omega t$ for $k$ is from 1 to 6 and $t=80$ were obtained. Bi-variate correlation analysis confirmed the independence of series $sink\omega t$ and $cos k\omega t$. Then $R_t$ was regressed on them, fitted Circular Model is;

\[
R_t = 0.34407\cdot 2.5198\cos \omega t
\]  

(12)

RMSE is 3.9 in model fitting and 1.8 in forecasting. Residuals were normally distributed and independent. Therefore Circular Model is suitable in forecasting returns of the company.

Same procedures were followed for rest of the companies in the sample. Summary of the analysis given in Table 2;

<table>
<thead>
<tr>
<th>Company</th>
<th>ARIMA</th>
<th></th>
<th>Circular Model</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RMSE</td>
<td>RMSE</td>
<td>RMSE in Model Fitting</td>
<td>RMSE in Forecasting</td>
</tr>
<tr>
<td></td>
<td>in Model Fitting</td>
<td>in Forecasting</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HUNA</td>
<td>7.2</td>
<td>3.6</td>
<td>3.9</td>
<td>1.8</td>
</tr>
<tr>
<td>SIGI</td>
<td>7.5</td>
<td>4.6</td>
<td>7.4</td>
<td>4.0</td>
</tr>
<tr>
<td>PALM</td>
<td>7.4</td>
<td>8.4</td>
<td>8.6</td>
<td>8.8</td>
</tr>
<tr>
<td>ACME</td>
<td>8.4</td>
<td>9.3</td>
<td>8.2</td>
<td>9.6</td>
</tr>
<tr>
<td>TOKY</td>
<td>6.7</td>
<td>5.8</td>
<td>6.6</td>
<td>5.9</td>
</tr>
<tr>
<td>BLUE</td>
<td>9.7</td>
<td>9.3</td>
<td>8.8</td>
<td>9.6</td>
</tr>
<tr>
<td>BOGAL</td>
<td>8.8</td>
<td>8.4</td>
<td>7.4</td>
<td>7.7</td>
</tr>
<tr>
<td>LFIN</td>
<td>8.1</td>
<td>6.3</td>
<td>7.8</td>
<td>5.7</td>
</tr>
<tr>
<td>DFCC</td>
<td>8.4</td>
<td>6.5</td>
<td>8.3</td>
<td>6.0</td>
</tr>
<tr>
<td>HASU</td>
<td>6.6</td>
<td>5.5</td>
<td>6.7</td>
<td>5.7</td>
</tr>
<tr>
<td>AGAL</td>
<td>8.2</td>
<td>6.8</td>
<td>8.8</td>
<td>7.8</td>
</tr>
<tr>
<td>BALA</td>
<td>8.7</td>
<td>8.2</td>
<td>8.5</td>
<td>8.2</td>
</tr>
<tr>
<td>BOGA</td>
<td>9.6</td>
<td>7.6</td>
<td>9.3</td>
<td>8.6</td>
</tr>
<tr>
<td>WATA</td>
<td>7.6</td>
<td>7.2</td>
<td>7.9</td>
<td>6.6</td>
</tr>
<tr>
<td>CLAND</td>
<td>8.0</td>
<td>6.5</td>
<td>7.8</td>
<td>6.9</td>
</tr>
<tr>
<td>PDL</td>
<td>5.9</td>
<td>5.4</td>
<td>5.7</td>
<td>5.2</td>
</tr>
<tr>
<td>EQIT</td>
<td>7.9</td>
<td>8.7</td>
<td>7.9</td>
<td>9.1</td>
</tr>
<tr>
<td>BREW</td>
<td>8.3</td>
<td>6.5</td>
<td>8.3</td>
<td>6.0</td>
</tr>
<tr>
<td>JKH</td>
<td>7.4</td>
<td>5.2</td>
<td>7.3</td>
<td>4.5</td>
</tr>
<tr>
<td>LANK</td>
<td>10.3</td>
<td>8.5</td>
<td>10.5</td>
<td>8.8</td>
</tr>
</tbody>
</table>

Results revealed that the RMSE is sufficiently small in model fitting and model verification for both ARIMA and CM. Also forecasting errors are almost same in both methods. Residuals of all the models were normally distributed and uncorrelated.

However, Forecasted values of ARIMA models follow linear patterns, not the patterns of actual returns. These results agree with the findings of Ayodele et al. (2015). But forecasts of CM follow the patterns of actual returns. For an example, Figure 2 clearly shows the difference in patterns of ARIMA forecasts and CM forecast for the company HUNA;
Figure 2: Actual Vs ARIMA Forecasts & CM Forecasts

IV. CONCLUSION

Scientific forecasting plays a vital role in financial markets. Statistical techniques and soft computing techniques are used for forecasting share prices or share returns. ARIMA model is a well known univariate statistical model served for the purpose. Circular Model (CM) is a newly introduced model for forecasting share returns. Both models are appropriate when a data series follow a wave like pattern.

This study was focused to compare the forecasting ability of ARIMA and CM in Sri Lankan context. Results confirmed the forecasting ability of both the models. However, ARIMA forecasts do not follow the patterns of actual returns, while CM forecasts follow them. It was concluded that CM is superior to ARIMA in forecasting Sri Lankan stock returns.

It is recommended to extend the study for the other share markets.

REFERENCES


