

Experiments of Randomized Hints on an Axiom of Infinite-Valued Lukasiewicz Logic

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Abstract: In this paper, we present an experiment of our randomized hints strategy of automated reasoning for yielding Axiom(5) from Axiom(1)(2)(3)(4) of Infinite-Valued Lukasiewicz Logic. In the experiment, we randomly generated a set of hints with size ranging from 30 to 60 for guiding hyper-resolution based search by the theorem prover. We have successfully found the most useful hints list (with 30 clauses) among 150 * 6 hints lists. Also, we discuss a curious non-linear increase of generated clauses in deducing Axiom(5) by applying our randomized hints strategy.

Keywords: Randomized hints, Lukasiewicz Logic, hyper-resolution, condensed detachment, OTTER.

I. LUKASIEWICZ MANY-VALUED LOGIC

Infinite-valued logic is the classic and promising field in automated reasoning. Recently, Pykacz discussed whether quantum mechanics should be described by many-valued logic [1]. Infinite-valued logic can cope with the world without the excluded middle of which principle Kolmogorov discussed in detail [2]. Pablo et al. [3] point out that Kleene's many-valued logic is used to cope with sentences such as Liar or the Truth-Teller. In the experiment, we use the automated reasoning program OTTER [4]. We apply a randomized hints strategy [10] for directing a hyper-resolution based search.

II. BODY OF ARTICLE

A. The semantics of infinite value logic

Lukasiewicz firstly presented the semantics of the infinite-valued logic (sentential calculus). His semantics aims to interpret the sentential variables that appeared in Principia Mathematics by Whitehead and Russell [5]. We call the axiomatics of Lukasiewicz logic corresponding to Harris et al. [6].

We introduce a predicate v for assigning rational numbers with the closed unit interval $[0,1]$. The interval $[0,1]$ is applied to each atomic sentence in the Lukasiewicz axiom. Using v , we can cope with the sentences determined recursively, as follows.

$$\rightarrow: v(C_{pq}) = p \rightarrow q = \begin{cases} 0 & \text{if } p \geq q \\ q - p & \text{if } p < q \end{cases} \quad (1)$$

$$\&: v(K_{pq}) = \max(p, q) \quad (2)$$

$$\vee: v(A_{pq}) = \min(p, q) \quad (3)$$

$$\neg: 1 - p \quad (4)$$

Here in (1), the predicate v represents a conditional of the C_{pq} , which means p implies q . In (2), v assigns a conjunction K_{pq} which yields the maximum of the values of p and q . In (3), A_{pq} which is V means a disjunction by the minimum of the values of p and q . Material implication of Lukasiewicz logic truth table is as follows:

TABLE I
ŁUKASIEWICZ LOGIC TRUTH TABLE.

$A \rightarrow B$		B		
		F	U	T
A	F	T	T	T
	U	U	T	T
	T	F	U	T

The Lukasiewicz logic differs from Kleene logic in its definition of the implication that "unknown implies unknown" is true.

B. The Axiomatics of Hilbert-style system for infinite-valued logic

Lukasiewicz has conjectured that the set of five axiom schemes (A1-A5) and the single rule of condensed detachment (CD) [11]. With A1-A5 and CD, the semantics of Lukasiewicz are completed.

- (A1) : $A \rightarrow (B \rightarrow A)$ (5)
- (A2) : $((A \rightarrow B) \rightarrow (B \rightarrow C)) \rightarrow (A \rightarrow C)$ (6)
- (A3) : $((A \rightarrow B) \rightarrow B) \rightarrow ((B \rightarrow A) \rightarrow A)$ (7)
- (A4) : $(\neg A \rightarrow \neg B) \rightarrow (B \rightarrow A)$ (8)
- (A5) : $((A \rightarrow B) \rightarrow (B \rightarrow A)) \rightarrow (B \rightarrow A)$ (9)

- (CD) $Cpq : \{p \rightarrow q, p\} \rightarrow q$
- (A1) $CpCqp$
- (A2) $CCpqCCqrCpr$
- (A3) $CCCpqqCCqpp$
- (A4) $CCNpNqCqp$
- (A5) $CCCpqCqpCqp$

It is said that Wajsberg gave a proof of this conjecture in the 1930s. But it has been lost (not published). Afterward, in the 1950s, Rose and Rosser [7] provided the proofs. Also, the completeness of the proofs concerning more general classes of the algebraic structure was presented in [8].

III. METHODOLOGY

A. Hyper-Resolution

Hyper-Resolution takes a non-positive clause called the nucleus and simultaneously infers each of its negative literals. Those negative literals are called satellites. The general scheme is:

$$\begin{aligned}
 &K_{1,l}, \dots, K_{1,n} \\
 &\dots \\
 &K_{m,l}, \dots, K_{m,n} \\
 &\frac{\{\neg L_1, \dots, \neg L_{m+1}, L_l\} \exists \sigma.\sigma}{= \text{mgu}(\{|K_{1,l}, \dots, K_{m,l}\}, \{|L_1, \dots, L_m\})} \\
 &\{K_{1,2}, \dots, K_{1,n}, K_{m,2}, \dots, K_{m,n}, L_{m+1}, \dots, L_l\} \sigma
 \end{aligned}$$

$K[i]$ denotes the clauses in the list above, and $L[i]$ denotes the literal. Hyper-Resolution is applied to a set of m unit clauses $\{K_1\} \dots \{K_m\}$ and a single nucleus $\{L_1, \dots, L_{m+1}\}$ consisting of $m + 1$ literals.

B. Condensed detachment

Condensed detachment [9] enables us to combine substitutions and modus ponens into a single rule.

$i(s,t)$: major premise

r : minor premise

yields t_δ , where δ is a most general unifier for terms r and s . For the expression of condensed detachment in the language of OTTER, we apply hyperresolution. The single rule of condensed detachment is a particular instance of a nucleus of the form:

list(usable).

$\neg P(i(x,y)) \mid \neg P(x) \mid P(y)$.

end_of_list.

Here the predicate P (provable) is a unary predicate symbol. If we apply condensed detachment to equation (1), we obtain clause 61 as follows.

1 [] $\neg P(i(x,y)) \mid \neg P(x) \mid P(y)$.

2 [] $P(i(x,i(y,x)))$.

61 [hyper,1,2,2] $P(i(x,i(y,i(z,y))))$.

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C. Hints: a proof sketch

Hints in OTTER can be described as a proof sketch. The proof sketch for yielding A5 from A1-A4 can be thought of as a set of conditions which is lemmas to prove sufficient for the proof. Ideally, a proof sketch composed of a hints list is already a proof. Then the conditions are also met. The hints strategy is closely related to the weighting strategy. Both strategies provide the desired control over the reasoning process.

Weighting focus on each clause for assigning a weight by upper-supplied weight templates. Templates provide a recursive mapping of weight from term and atoms corresponding the values of weight as follows:

```
weight_list(pick_given).
weight(P(i(x,i(y,x))), 2).
weight(P(i(i(x,y),i(i(y,z),i(x,z))))), 2).
end_of_list.
```

In contrast, the hints strategy copes with the identification of key clauses in the reasoning process instead of the general calculation of weights.

```
list(hints).
P(i(i(x,y),i(i(z,x),i(z,y))))
end_of_list.
```

The hint strategy enhances the weighting strategy in two points.

1. Hints strategy focuses on entire facts (clauses) rather than terms and subterms.
2. Hints strategy uses subsumption for determining the value of a generated clause.

The hints strategy provides both semantic and logical components for the evaluation of a generated clause by leveraging subsumption.

IV. EXPERIMENT

In experiment, we use workstation with Intel(R) Xeon(R) CPU E5-2620 v4 (2.10GHz) and 251G RAM. We set the list of set of support corresponding to the equations (1)-(4) as follows:

```
list(sos).
P(i(x,i(y,x))).
P(i(i(x,y),i(i(y,z),i(x,z))))).
P(i(i(i(x,y),y),i(i(y,x),x))).
P(i(i(n(x),n(y)),i(y,x))).
end_of_list.
```

We set the passive list to search unit conflict to yield the proof from (1)-(4) to (5).

```
list(passive).
-P(i(i(i(a,b),i(b,a)),i(b,a))).
end_of_list.
```

Figure 1 shows the average size of the set of support. In the experiment, we repeated the random extraction of hints 150 times. Average is calculated for each size of the hints list. Also, Figure 2 shows the average number of generated clauses.

$$\text{average size} = \frac{\sum_{n=1}^{150} X_n}{150}$$

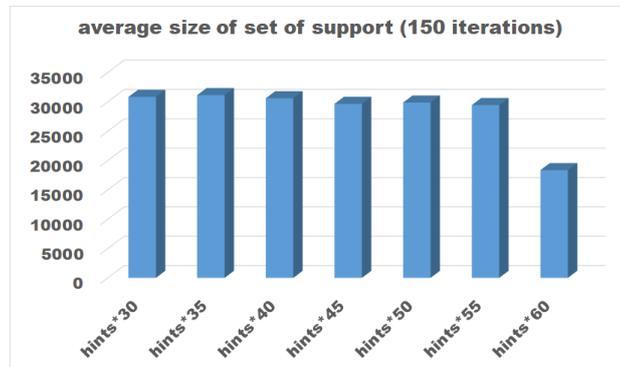


FIGURE: 1

The X-axis of Figure 1 and Figure 2 is the size of the hints list, ranging from 30 to 60. The size of set support shown in Figure 1 is relatively stable compared with Figure 2. The number of generated clauses is drastically increased between hints*40 to hints*45.

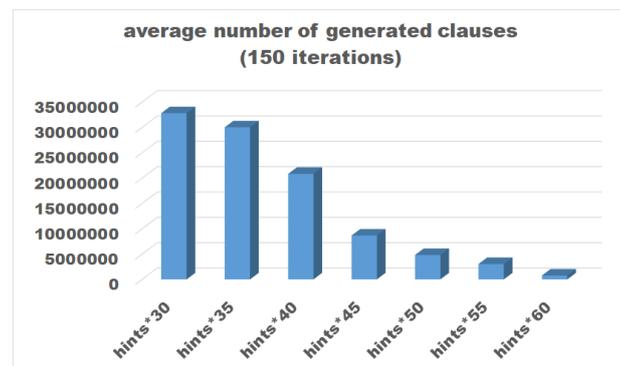


FIGURE: 2

Figure 3 depicts the size of the set of support for every 150 times of iterations. We sort the X-axis by the size of the set of support. Figure 4 depicts the number of generated clauses for every 150 times of iterations. We sort the X-axis by the number of generated clauses. In Figure 3, some change points are observed in areas A (x=95-100) and B (x=126-129). Curiously, there is no difference between hints*30 and hints*45 from x=1 to 95. In Figure 4, we observe two change points, A (x=84-87) and B (x=119-120). During these periods, the number of generated clauses with hints*30 and hints*45 are drastically increased.

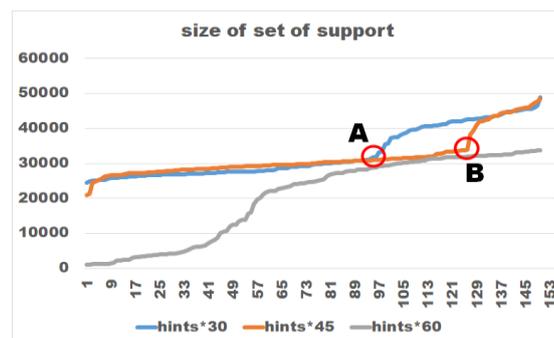


FIGURE: 3

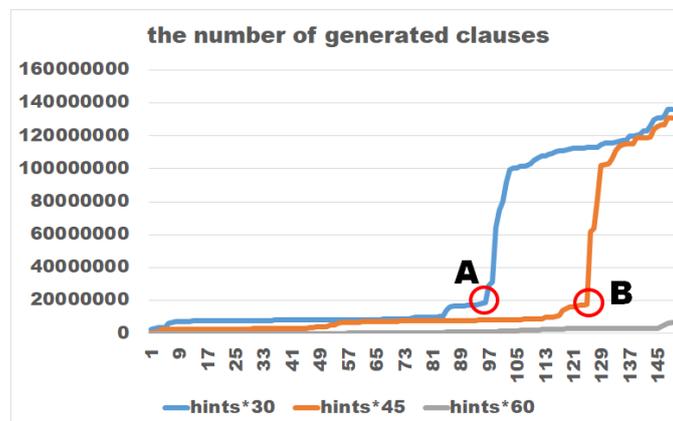


FIGURE: 4

We use the automated reasoning program OTTER [4]. Veroff present the effectiveness of hints strategy in automated reasoning [10]. The axiomatics of L0 is presented in [6]. The use of condensed detachment is discussed in [11].

V. CONCLUSION

In this paper, we present an experiment of our randomized hints strategy of automated reasoning for yielding Axiom 5 on Lukasiewicz axiom of infinite valued logic. We have successfully found the most useful 30 hints among 900 hints lists. Also, we have observed some curious non-linear increases in the size of set of support and the number of generated clauses in deducing Axiom (5) by comparing the number of hints from 30 to 60.

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