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Forecasting Conditional Volatility of Inflation Rate in Nigeria Using Artificial Neural Networks

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Abstract: **Forecasting volatility is an essential step in many financial decision makings. GARCH family of models has been extensively used in finance and economics, particularly for estimating volatility. The motivation of this study is to enhance the ability of GARCH models in forecasting the return volatility. A hybrid model was proposed based on EGARCH and Artificial Neural Networks to forecast the volatility of Inflation rate in Nigeria. The estimates of volatility obtained by an EGARCH model are fed forward to a Neural Network. The input to the hybrid model is complemented by historical values of other explanatory variables. The forecasts obtained by the hybrid model have been compared with those of EGARCH model in terms of closeness to the standard deviation of the Monthly returns. The computational results demonstrate that the hybrid model provides better volatility forecasts.**

Keywords: **Forecasting volatility, Artificial Neural Networks (ANNs), EGARCH model.**

1. INTRODUCTION

Artificial Neural Networks (ANNs) are non-linear data driven self adaptive approach as opposed to the traditional model based methods. They are powerful tools for modeling, especially when the underlying data relationship is unknown. ANNs can identify and learn correlated patterns between input data sets and corresponding target values. After training, ANNs can be used to predict the outcome of new independent input data. ANNs imitate the learning process of the human brain and can process problems involving non-linear and complex data even if the data are imprecise or noisy. Thus they are ideally suited for the modeling of financial data which are known to be complex and often non-linear.

When discussing the volatility of time series, econometricians refer to the 'conditional variance' of the data and the time-varying volatility typical of asset returns is otherwise known as 'conditional heteroscedasticity'. The concept of conditional heteroscedasticity was introduced to economists by Engle (1982), who proposed a model in which the conditional variance of a time series is a function of past shocks; the autoregressive conditional heteroscedastic (ARCH) model. The model provided a rigorous way of empirically investigating issues involving the volatility of economic variables. An example is Friedman's hypothesis that higher inflation is more volatile (Friedman, 1977). Using data for the UK, Engle (1982) found that the ARCH model supported Friedman's hypothesis. Engle (1983) applied the ARCH model to US inflation and the converse results emerged, although Cosimano and Jansen (1988) criticize this paper as they believe that Engle estimates a mis-specified model. The relationship between the level and variance of inflation has continued to interest applied econometricians (see, for example, Grier and Perry, 2000).

Artificial Neural Network (ANN) provides a flexible way of examining the dynamics of various economic and financial problems. The application of ANN to modeling economic conditions is expanding rapidly (Hamid & Iqbal, 2004; Kim, 2006; Wang, 2009; Yu, Wang, & Keung, 2009). Bildirici and Ersin (2009) enhanced ARCH/GARCH family of models with ANN and used them in order to forecast the volatility of daily return in Istanbul Stock Exchange. Roh (2007) proposed three hybrid time series and ANN models for forecasting the volatility of Korea Composite Stock Price Index (KOSPI 200). Two of the most important applications of GARCH models in finance are forecasting and simulation. The power of a model in forecasting volatility is highly important since volatility is an essential input to many financial decision making models. The motivation of this study is to enhance the ability of GARCH models in forecasting inflation rate return in Nigeria volatility.

There has been growing interests in the time series modeling of financial data with changing variance over time in Nigeria. These parametric models for financial asset volatilities have gone through major developments since the original Autoregressive Conditional Heteroscedasticity (ARCH) and Generalized ARCH (GARCH) models introduced by Engle (1982) and Bollerslev (1986). These models have been extensively used in Nigerian financial Markets (see for example, Musa *et al* (2014), Amaefula and Asare, (2014), Salisu, and Mobolaji ,(2013), and Bala and Asemota,(2013)). Such time series models with heteroscedastic errors are specifically to modeling financial market data which are highly volatile. Although, many financial time series observations have non-linear dependence structure, a linear correlation structure is usually assumed among the time series data. Therefore, ARCH type models may not capture such nonlinear patterns and linear approximation models of those complex problems may not be satisfactory. Nonparametric models estimated by various methods such as Artificial Intelligence (AI), can be fit on a data set much better than linear models.

2. GARCH-TYPE MODELS

The volatile behavior in financial markets is referred to as the ‘‘volatility’’. Volatility has become a very important concept in different areas of financial engineering, such as multi-period portfolio selection, risk management, and derivative pricing. Also, many asset pricing models use volatility estimates as a simple risk measure. In statistics, volatility is usually measured by standard deviation or variance (Daly, 2008). Recently, numerous models based on the stochastic volatility process and time series modeling have been found as alternatives to the implied and historical volatility approach. The most widely used model for estimating volatility is ARCH (Auto Regressive Conditional Heteroscedasticity) model developed by Engle in 1982. Since the development of the original ARCH model, a lot of research has been carried out on extensions of this model among which GARCH (Bollerslev, 1986), EGARCH (Nelson, 1991) and GJR-GARCH (Glosten, Jagannathan, & Runkle, 1993) are the most frequently used models. Generalized Autoregressive Conditional Heteroscedasticity, GARCH (p, q), is a generalization of ARCH model by making the current conditional variance dependent on the p past conditional variances as well as the q past squared innovations. The GARCH (p, q) model can be written as:

$$\varepsilon_t = \sigma_t z_t \quad (1)$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{P} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \quad (2)$$

$\omega, \alpha_i, \beta_j$ are nonnegative coefficients, z_t represents a sequence of independent and identically distributed (i.i.d.) random variables with zero mean and unit variance. By definition, ε_t is a serially uncorrelated sequence with zero mean and the conditional variance of σ_t^2 which may be nonstationary. By accounting for the information in the lag(s) of the conditional variance in addition to the lagged ε_{t-i}^2 terms, the GARCH model reduces the number of parameters required. However, ARCH or GARCH models fail to capture asymmetric behavior of the returns. Exponential Generalized Autoregressive Conditional Heteroscedasticity (EGARCH) model was introduced by Nelson (1991) and Nelson and Cao (1992) to account for leverage effects of price change on conditional variance. The EGARCH model can be represented as follows:

$$log\sigma_t^2 = \omega + \sum_{j=1}^{P} \alpha_i \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} - E\left(\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right) \right| + \sum_{j=1}^{q} \beta_j log\sigma_{t-j}^2 + \sum_{k=1}^{r} \gamma_k \left(\frac{\varepsilon_{t-k}}{\sigma_{t-k}}\right) \quad (3)$$

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This model places no restrictions on the parameters α_i and β_j to ensure nonnegative of the conditional variances. In addition to the EGARCH model, another model for capturing the asymmetric features of returns behavior is GJR-GARCH model. The GJR model is closely related to the Threshold GARCH (TGARCH) model proposed by Zakoian (1994) and the Asymmetric GARCH (AGARCH) model of Engle (1990). The GJR model of Glosten et al. (1993) allows the conditional variance to respond differently to past negative and positive innovations. The GJR (p, q) model may be expressed as:

$$\sigma_t^2 = \omega + \sum_{i=1}^{P} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{P} [\gamma_i d(\varepsilon_{t-i} < 0)\varepsilon_{t-i}^2] + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \quad (4)$$

Where $\gamma_i (i = 1, \ldots, P)$ are the asymmetric parameter and $d(.)$is the indicator function defined such that $d((\varepsilon_{t-i} < 0) = 1$ if $\varepsilon_{t-i} < 0$ and $d((\varepsilon_{t-i} > 0) = 0$ if $\varepsilon_{t-i} > 0$

Similarly, Ding, *et al.* (1993) introduced another asymmetric model APARCH (*p*, *q*) which is written as:

$$\sigma_t^\delta = \omega + \sum_{j=1}^{P} \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^\delta + \sum_{j=1}^{q} \beta_j log \sigma_{t-j}^\delta \quad (5)$$

where the asymmetric parameter $-1 < \gamma_i < 1 (i = 1 \ldots, P), \delta$ is the non-negative Box-Cox power transformation of the conditional standard deviation process and asymmetric absolute innovations. This power parameter is estimated along with other parameters in the model. In this study, we use three penalized model selection criteria, the Hannan-Quinn criteria, Bayesian information criterion (BIC) and Akaike's information criterion (AIC), to select best lag parameters for GARCH models (Akaike, 1973; Schwarz, 1998).

3. A MULTI-LAYER PERCEPTRON ANNS

The underlying structure of an MLP is a directed graph, i.e. it consists of vertices and directed edges, in this context called neurons and synapses. The neurons are organized in layers, which are usually fully connected by synapses. In MLP, a synapse can only connect to subsequent layers. The input layer consists of all covariates in separate neurons and the output layer consists of the response variables. The layers in between are referred to as hidden layers, as they are not directly observable.

To each of the synapses, a weight is attached indicating the effect of the corresponding neuron, and all data pass the neural network as signals. The signals are processed first by the so-called integration function combining all incoming signals and second by the so-called activation function transforming the output of the neuron. The simplest multi-layer perceptron (also known as perceptron) consists of an input layer with n covariates and an output layer with one output neuron. It calculates the function

$$O(X) = f(\omega_0 + \sum_{i=1}^{n} \omega_i x_i) = f(\omega_0 + \omega^T X) \quad (6)$$

Where ω_0 denotes the intercept, $\omega = (\omega_1, \ldots, \omega_n)$ the vector consisting of all synaptic weights without the intercept, and $X = (x_1, x_2, \ldots, x_n)$ the vector of all covariates. The function is mathematically equivalent to that of Generalized Linear Model (GLM) with link function f^-. Therefore, all calculated weights are in this case equivalent to the regression parameters of the GLM. Such an MLP with a hidden layer consisting of J hidden neurons calculates the following function:

$$O(X) = f(\omega_0 + \sum_{j=1}^{J} w_{j.} h_j) = f(\omega_0 + \sum_{j=1}^{J} \omega_j . f(\omega_{0j} + \sum_{i=1}^{n} \omega_{ij} x_i)) \quad (7)$$

Where $h_j = f(w_{0j} + \sum_{i=1}^{n} w_{ij} x_i), j = 1,2 \ldots J$

$$O(X) = f(\omega_0 + \sum_{j=1}^{J} \omega_j . f(\omega_{oj} + \omega_j^T X)) \quad (8)$$

Where ω_0 denotes the intercept of the output neuron and ω_{0j} the intercept of the j_{th} hidden neuron. Additionally, ω_j denotes the synaptic weight corresponding to the synapse starting at the j_{th} hidden neuron and leading to the output neuron, $\omega_j = (\omega_{1j}, \ldots, \omega_{nj})$ the vector of all synaptic weights corresponding to the synapses leading to the j_{th} hidden neuron, and $X = (x_1, \ldots x_n)$ the vector of all covariates.

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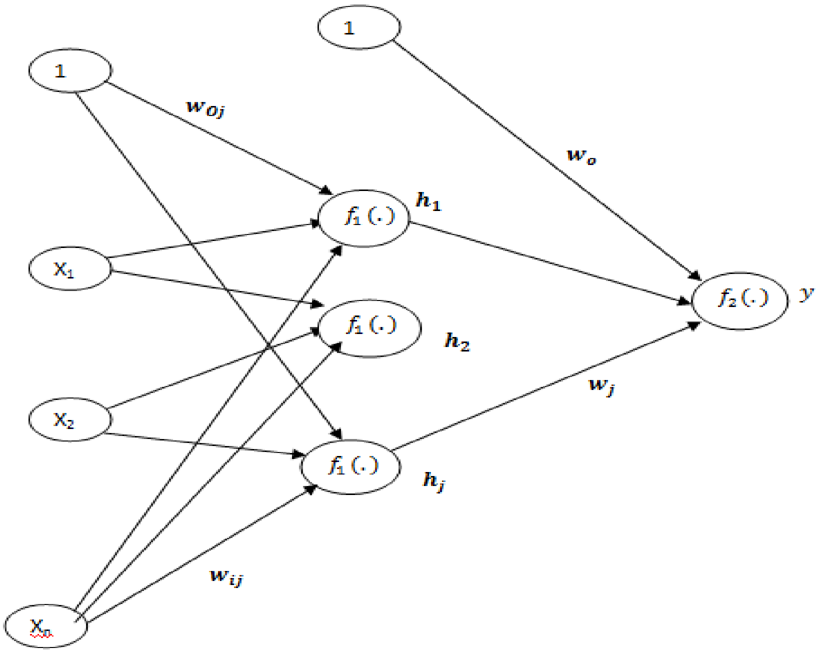

Figure 1 the Architecture of multi layer feed forward neural network with n covariates

In this study, we apply a resilient back propagation Neural Network which mostly used when there is large datasets with associated predictors. One of the problems that happen during ANN training is over fitting. "Early stopping" is a technique based on dividing datasets in to three subsets: training set, testing set, and hold-out set. The training set is used for computing the gradient and updating the network weight and biases. The error on the testing set is screened during the training process. When the testing error increases over a specified number of iterations, the training is stopped, and the weights and biases are returned. In this study, the dataset is divided as: 50% for training, 25% for testing and 25% for hold-out.

4. THE PROPOSED ANN MODEL

In this, we propose a hybrid model for forecasting conditional volatility of the inflation rate in Nigeria. Initially, a preferred GARCH model is identified upon which the hybrid model is built. For this reason, optimum lags for GARCH model is estimated using AIC, BIC and HQC indices. Then, the model is used for predicting some months ahead and the preferred model is selected according pre-defined measures. Next, we explain how the selected GARCH model is hybridized with ANN models.

The underlying concept for the hybrid model is that there are some explanatory factors other than historical prices that affect the future prices of inflation rate in the market. We forecast the volatility of inflation rate in Nigeria with a number of market variables which affect the inflation rate returns.

Selection of the input variables depends on the knowledge of which ones affect the volatility significantly. We have chosen 5 endogenous variables related to inflation rate and 6 exogenous variables. The endogenous variables are inflation rate prices, squared inflation rate prices, inflation rate returns, squared inflation rate returns and volatility (based on the preferred model). The exogenous variables include all share indexes (ASI), crude oil prices (COP), Naira/US Exchange rate (EXRT), interest rate (INRT) and Money Supply (MSPLY) of the Nigerian Financial market. The final set of selected variables contains 8 explanatory variables which have significant correlations with the estimated volatility based on the preferred GARCH model. The selected variables are shown in table 1.

To this stage, the input variables to the ANN have been specified. Next, the standard deviation is considered to be the target outputs for training the network. Figure 2 shows the flowchart of this modeling process and Figure 3 is Schematic representation of the hybrid model

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Table 1: Selected explanatory variables

1	inflation rate price
2	squared inflation rate price
3	inflation rate returns
4	squared inflation rate returns
5	volatility (based on preferred model)
6	crude oil price
7	Naira/US Dollar exchange rate
8	Interest rate price

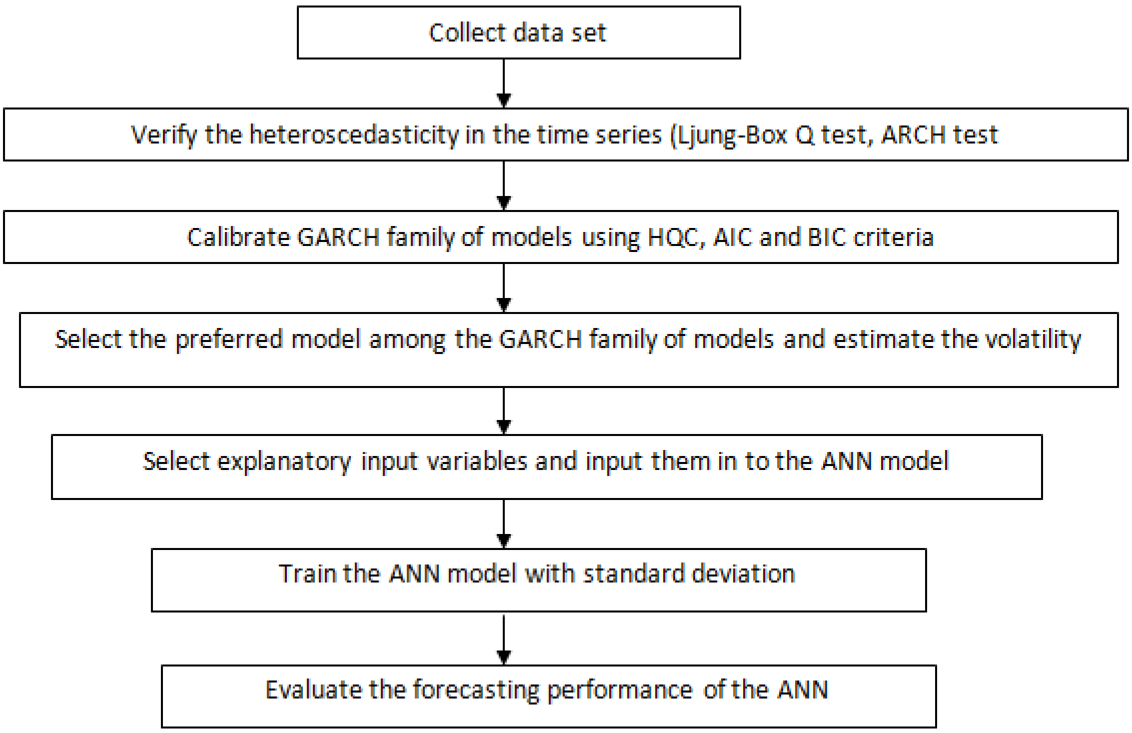

Figure 2: Flowchart of modeling process of ANN for inflation rate

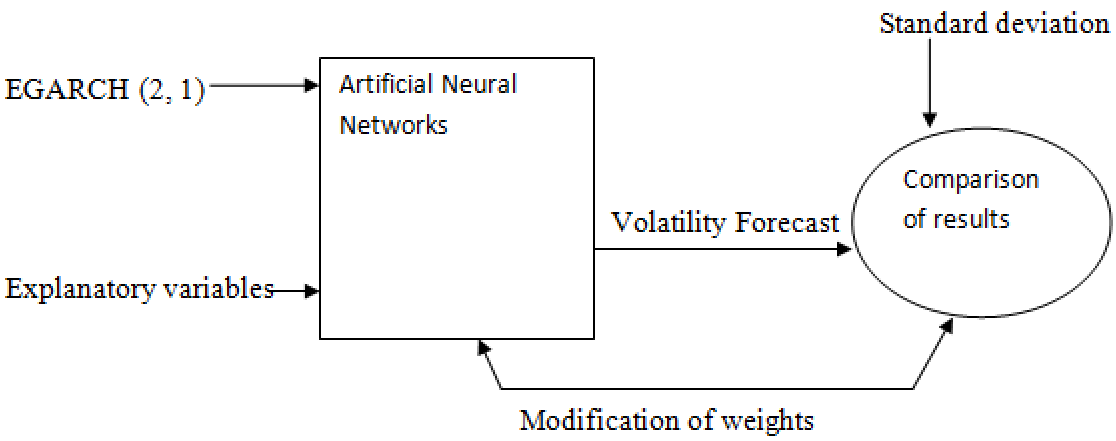

Figure3: Schematic representation of ANN model for inflation rate

5. NIGERIAN INFLATION RATE CHARACTERISTICS

This study used Monthly prices of inflation rate in Nigeria over a period of January 1995 to February, 2015. All the data were gathered from the Central Bank of Nigeria through their website www.cbn.gov.ng. The logarithmic returns of the series were calculated. Table 2 shows

Table 2 Data description and preliminary statistics of the inflation rate reruns in Nigeria

Mean	0.998872
Standard Deviation	2.085198
Skewness	0.777447
Kurtosis	9.117109
Jarque-Bera	400.0271(0.000)*
Observation	242
$Q^2(15)^a$	23.783(0.049)*
ARCH test (15)[b]	38.48665(0.0008)*

[a] is the Ljung-Box Q test for the 15th order serial correlation of the squared returns

[b] Engle's ARCH test also examines for autocorrelation of the squared returns.

- Significantly at the 5%.

The basic statistical characteristics of the return series. The statistics showed excess kurtosis since the kurtosis of the returns exceeds the normal value of 3. In terms of skewness, Inflation rate returns are positively skewed distribution. This therefore indicates that the returns series have fat tails and sharper peaks than the normal distribution. The Jarque-Bera normality test confirms non-normality of the distributions of the variable. Also, the Ljung-Box $Q^2(15)$ statistics and Engle's ARCH test for the squared returns indicate that the return series exhibit linear dependence and strong ARCH effects. Thus, the preliminary analysis of data suggests the use of GARCH models to capture fat-tails and time varying volatility in such series.

Figure 4 and 5 show the monthly of inflation rate price and its logarithmic returns. Informally, Fig.4 suggests that the inflation is trending or non stationary. Fig.5 presents inflation rate returns. Generally inflation rate returns concentrate around zero with largest changes between June and July of the year 2003, moderate changes in the year 2001. These changes are more consistent (concentrated around zero) from 2009 to 2015

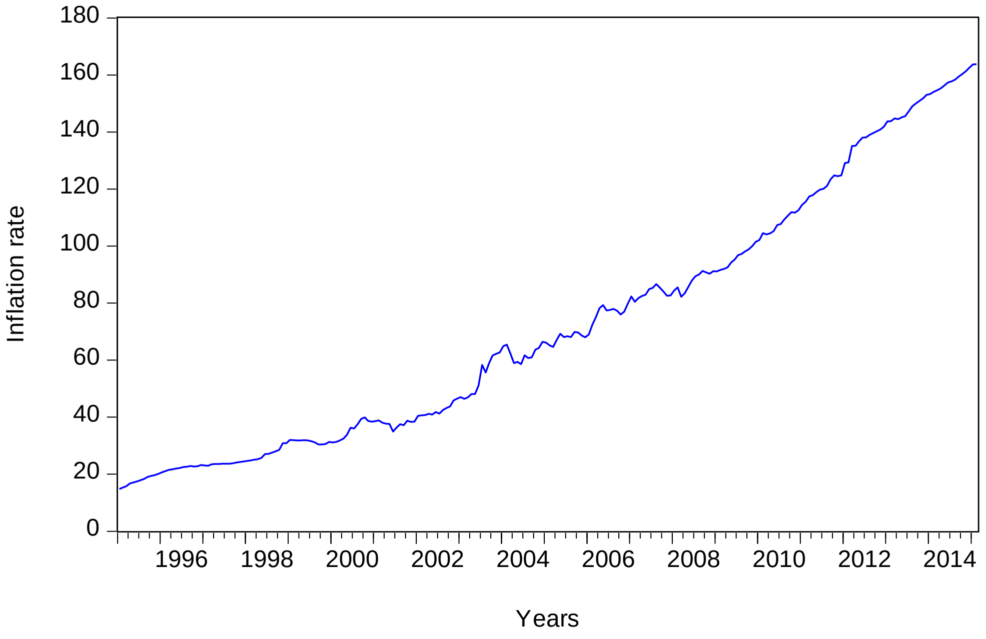

Figure 4 Plot of Monthly Inflation rate in Nigeria from 1995-2015

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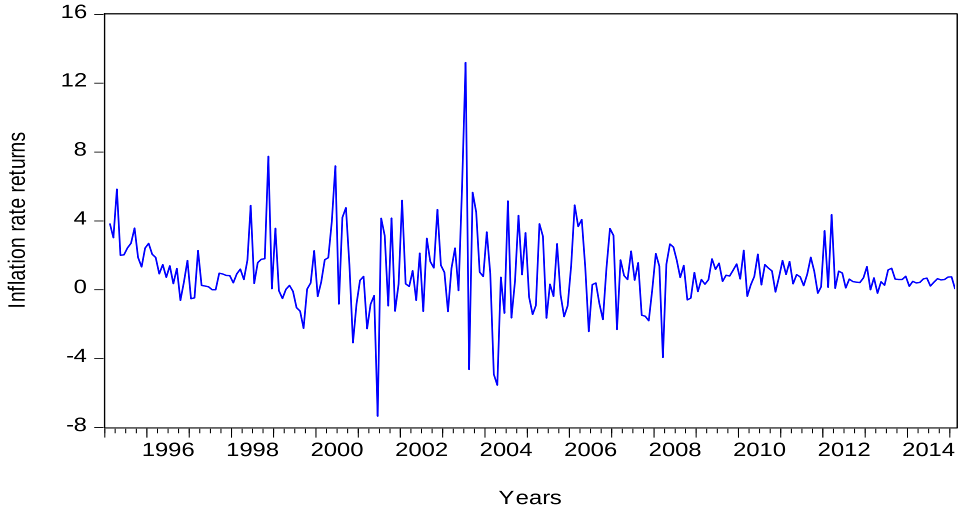

Figure5 Plot of Monthly inflation rate returns in Nigeria from 1995-2015

To evaluate forecast accuracy, this study compares the volatility forecasts of the proposed hybrid models with the monthly standard deviation as a measure of the actual volatility. The monthly standard deviation on month t is calculated by

$$SD_t = \sqrt{n^{-1}\sum_{i=t}^{t+n}(R_i - \bar{R})^2}, \; (9)$$

Where R_i is logarithmic return, $\bar{R} = \sum_{i=1}^{n} {R_i}/{n}$, and Where R_i is logarithmic return, $\bar{R} = \sum_{i=1}^{n} {R_i}/{n}$, and n is the number of months before nearest expiry option.

In addition, three measures are used to evaluate the performance of models in forecasting volatility as follows: root mean square error (RMSE), mean absolute error (MAE), and mean absolute percentage error (MAPE). These measures are defined as:

$$RMSE = \left(n^{-1}\sum_{i=1}^{n}(SD_i - \sigma i)^2\right)^{1/2}, \; (10)$$
$$MAE = n^{-1}\sum_{i=1}^{n}|SD_i - \sigma i|, \; (11)$$
$$MAPE = n^{-1}\sum_{i=1}^{n}\left|\frac{SD_i - \sigma i}{SD_i}\right| * 100, \; (12)$$

6. COMPUTATIONAL RESULTS

In this section, we present the results of applying GARCH-type models as well as the hybrid models for forecasting volatility of inflation rate returns in Nigeria. As the first step, GARCH, GJR-GARCH, EGARCH and APARCH models with various combinations of (p,q) parameters ranging from (1,1) to (3,3) were calibrated on historical return data. The best model turned out to be EGARCH (2, 1) according to AIC, BIC and HQC criteria. Table 3 shows AIC, BIC and HQC values for four GARCH-type models with best combinations of (p, q).

Table 3 Calibration of GARCH Models Using information selection criteria

Variables	GARCH Models	Selection criteria	
Inflation rate log returns	GARCH(3,2)	AIC	3.95288
		BIC	4.249342
		HQC	4.076159
	GJR-GARCH (3,3)	AIC	4.009518
		BIC	4.328577
		HQC	4.138076
	EGARCH(2,1)	**AIC**	**3.674033**
		BIC	**3.934309**
		HQC	**3.778893**
	APARCH(2,1)	AIC	3.959333
		BIC	4.249386
		HQC	4.076203

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According to the values of these criteria, EGARCH (2, 1) has shown the best performance and thus selected for the construction of the hybrid models.

To examine the fitness of the hybrid model and the selected EGARCH model, both of them have been used to forecast the volatilities for 62 months ahead and the results are reported in table 4.

Table 4 Models performance for 62 out of sample forecast of inflation rate returns volatilities

Measures	EGARCH(2,1)	Hybrid model I (8-4-1)
RMSE	1.072292	0.548286
MAE	0.830539	0.376029
MAPE	315.0673	178.9924

According to the values of fitness measures, the hybrid model outperform EGARCH model. This is confirmed in figure below

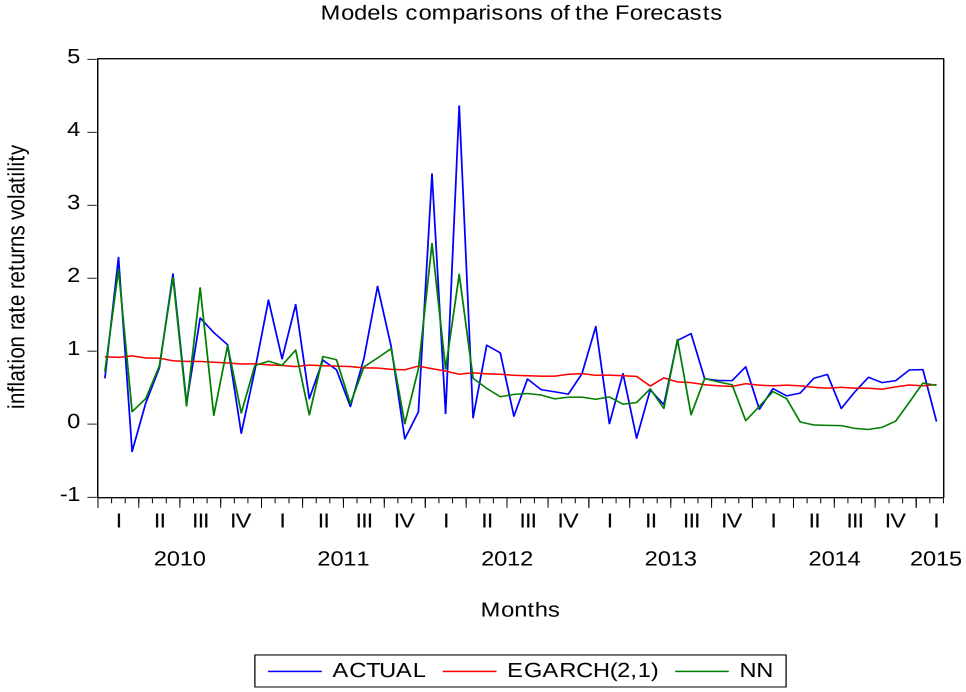

Figure 6 Comparison of Forecast performance of the EGARCH and NN Models with the Actual Volatility.

7. CONCLUSION

This research is based on the application of GARCH models. One of the limitations is that these models produce better results in relatively stable markets and could not capture violent volatilities and fluctuations. Therefore, it is recommended that these models be combined with other models when applied to violent markets as also suggested by Gourieroux (1997).

In this study, the problem of modeling and forecasting volatility of monthly inflation rate in Nigeria has been investigated. Four types of models from GARCH family have been calibrated and used for forecasting the volatility. Then, their performances have been compared according to pre-defined measures. The best model turns out to be EGARCH (2, 1). To enhance the forecasting power of the selected model, a hybrid model has been constructed using Artificial Neural Networks. The inputs to the proposed hybrid models include the volatility estimates obtained by the fitted EGARCH model as well as other explanatory variables. The computational results on the data demonstrate that the hybrid model, using Artificial Neural Network, provides better volatility forecasts. This model significantly improves the forecasts over the ones obtained by the best EGARCH model.

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