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INVENTORY MODEL WITH STOCHASTIC DEMAND AND PARTIAL BACKLOGGING

PRAVESH KUMAR SHARMA

Abstract: **In the present study we have formulated an inventory model in which the demand is defined as a probability distribution and shortages are allowed which is partially backlogged. In this paper, a variation of the finite Inventory process model i.e., the classical Newsboy problem is attempted.**

Keywords: **Inventory, Stochastic, Backlogging.**

1. INTRODUCTION

Practically the demand which cannot be satisfied always from the inventory and it leaves the system stock-out. If the order quantity is larger than the realised demand, the items which are left over at the end of period are sold at a salvage value or disposed off. Hence, both the factors such as salvage and stock-out situations are equally important. If the demand is uncertain then it must be predicted and the continuous sources of uncertainty or stochastic demand, has a different impact on optimal inventory settings and prevents optimal solutions from being found in closed form. Notably, there are cases in which the probability distribution of the demand for new products is typically unknown because of a lack of historical information, and the use of linguistic expressions by experts for demand forecasting is often employed. The Assorted level of demand is viewed in form of a special class of inventory evolution known as finite inventory process.

In this paper, a variation of the finite Inventory process model i.e., the classical Newsboy problem is attempted.

2. ASSUMPTIONS AND NOTATIONS

h, C_1	-	The cost of each unit produced but not sold called holding cost.
d, C_2	-	The shortage cost arising due to each unit of unsatisfied demand.
R	-	Random variable denoting the demand.
x_0	-	The truncation point.
C	-	Total cost per unit time.
S	-	Supply level S and $\hat{S}$ is the optimal value.
R_1, R_2	-	A random variables denoting the demand for both the case when demand is less and more than the production, its PDF is given by (R_1) , $f(R_2)$.
$f(R_1, \theta_1)$	-	The probability distribution function when $R_1 \leq x_0$.
$f(R_2, \theta_2)$	-	The probability distribution function when $R_2 > x_0$.
$C_1(Q_i)$	-	The cost of each unit of Newspapers purchased for several individual demand but not sold called salvage loss.
$C_2(Q_i)$	-	The shortage cost arising due to each unit of unsatisfied individual demand of Newspapers.
S_i	-	Optimal supply size.
$f_1(Q_i, \theta_1)$	-	The probability distribution function when $Q_i \leq Q_0$.
$f_2(Q_i, \theta_1)$	-	The probability distribution function when $Q_i > Q_0$.

$\psi(Z), E(C)$	-	The expected total cost
$\psi(\hat{Z}), E(\hat{C})$	-	The optimal expected cost
$\psi_1(Z)$	-	Total holding cost
$\psi_2(Z)$	-	Total shortage cost
Q_i	-	Random variable denoting the several individual demands at the i^{th} location where $i = 1,2,\dots,n$
$E(Q_i)$	-	The expected several individual demand before the truncation point x_0, $E(Q_i) = \frac{1}{\theta_1}$ and after the truncation point is $E(Q_i) = \frac{1}{\theta_2}$

3. MATHEMATICAL MODEL

The shortage cost is assumed proportional to the area under the negative part of the inventory curve. If the total cost per unit time is C, then the cost incurred during the interval T is given as

$$CT = \begin{cases} h\left(S - \frac{R}{2}\right)T & , R \le S \\ \frac{h}{2} St_1 + \frac{d}{2}(R-S)t_2 & , R > S \end{cases} \quad (1)$$

The expected total cost given in Hanssman.F is as follows

$$E(C) = h\int_0^S \left(S - \frac{R}{2}\right) f(R)dR + h\int_S^\infty \frac{S^2}{2R} f(R)dR + d\int_S^\infty \frac{(R-S)^2}{2R} f(R)dR \quad (2)$$

Accordingly *SCBZ* property is defined by the *pdf* as

$$f(R) = \begin{cases} \theta_1 e^{-\theta_1 R_1} & ; R_1 \le x_0 \\ e^{x_0(\theta_2-\theta_1)}\theta_2 e^{-\theta_2 R_2} & ; R_2 > x_0 \end{cases} \quad (3)$$

where x_0 is constant denoting truncation point. The probability distribution function is denoted as $f(R_1,\theta_1)$ if $R_1 \le x_0$ and $f(R_2,\theta_2)$ if $R_2 > x_0$ (4)

Now a change to the model in form of cost function is that here in this model the cost function is denoted as $\psi(Z)$.Total cost incurred during the interval T given is given as

$$C(T) = \begin{cases} \varphi_1(Z-Q)T & , Q \le Z \\ \frac{\varphi_1}{2} Zt_1 + \frac{\varphi_2}{2}(Q-Z)t_2 & , Q > Z \end{cases} \quad (5)$$

Therefore the expected total cost function per unit time is given in the form

$$\psi(Z) = \varphi_1\int_0^Z (Z-Q)f(Q)dQ + \varphi_1\int_Z^\infty \frac{Z}{2}\frac{t_1}{T} f(Q)dQ + \varphi_2\int_Z^\infty \frac{(Q-Z)}{2}\frac{t_2}{T} f(Q)dQ \quad (6)$$

From the geometry it follows that $\frac{t_1}{T} = \frac{Z}{Q}$ and $\frac{t_2}{T} = \frac{Q-Z}{Q}$ (7)

The uncertainty here is related to a well known property called as SCBZ Property. SCBZ property which is defined in model 3.4 is applied in this model 3.5. Accordingly *SCBZ* property is defined by the PDF as

$$f(Q) = \begin{cases} \theta_1 e^{-\theta_1 Q} & ; Q \le Q_0 \\ e^{Q_0(\theta_2-\theta_1)}\theta_2 e^{-\theta_2 Q} & ; Q > Q_0 \end{cases} \quad (8)$$

where Q_0 is constant denoting truncation point. The probability distribution function is denoted as $f(Q,\theta_1)$ if $Q \le Q_0$ and $f(Q,\theta_2)$ if $Q > Q_0$ (9)

$$\psi(Z) = \varphi_1 k_1 + \varphi_1 k_2 + \varphi_2 k_3 \quad (10)$$

where $k_1 = \int_0^Z (Z-Q)f(Q)dQ$, $k_2 = \int_Z^\infty \frac{Z}{2}\frac{t_1}{T} f(Q)dQ$, $k_3 = \int_Z^\infty \frac{(Q-Z)}{2}\frac{t_2}{T} f(Q)dQ$ (11)

Now Differentiating (6)

$$\frac{d\psi(Z)}{dZ} = \varphi_1 \int_0^{Q_0} (Z-Q) f(Q,\theta_1) dQ + \varphi_1 \int_{Q_0}^{Z} (Z-Q) f(Q,\theta_2) dQ + \varphi_1 \int_Z^{Q_0} \frac{Z}{2}\frac{t_1}{T} f(Q,\theta_1) dQ + \varphi_1 \int_{Q_0}^{\infty} \frac{Z}{2}\frac{t_1}{T} f(Q,\theta_2) dQ + \varphi_2 \int_Z^{Q_0} \frac{(Q-Z)}{2}\frac{t_2}{T} f(Q,\theta_1) dQ + \varphi_2 \int_{Q_0}^{\infty} \frac{(Q-Z)}{2}\frac{t_2}{T} f(Q,\theta_2) dQ \quad (12)$$

$$\frac{d\psi(Z)}{dZ} = \varphi_1 \int_0^{Q_0} (Z-Q)\theta_1 e^{-\theta_1 Q} dQ + \varphi_1 \int_{Q_0}^{Z} (Z-Q) e^{Q_0(\theta_2-\theta_1)}\theta_2 e^{-\theta_2 Q} dQ + \varphi_1 \int_Z^{Q_0} \frac{Z}{2}\frac{t_1}{T}\theta_1 e^{-\theta_1 Q} dQ + \varphi_1 \int_{Q_0}^{\infty} \frac{Z}{2}\frac{t_1}{T} e^{Q_0(\theta_2-\theta_1)}\theta_2 e^{-\theta_2 Q} dQ + \varphi_2 \int_Z^{Q_0} \frac{(Q-Z)}{2}\frac{t_2}{T} \theta_1 e^{-\theta_1 Q} dQ + \varphi_2 \int_{Q_0}^{\infty} \frac{(Q-Z)}{2}\frac{t_2}{T} e^{Q_0(\theta_2-\theta_1)}\theta_2 e^{-\theta_2 Q} dQ \quad (13)$$

$$\frac{d\psi(Z)}{dZ} = \varphi_1 \left[\theta_1 \int_0^{Q_0} Z\, e^{-\theta_1 Q} dQ - \theta_1 \int_0^{Q_0} Q\, e^{-\theta_1 Q} dQ + \theta_2 \int_{Q_0}^{Z} Z e^{Q_0(\theta_2-\theta_1)} e^{-\theta_2 Q} dQ - \theta_2 \int_{Q_0}^{Z} Q e^{Q_0(\theta_2-\theta_1)} e^{-\theta_2 Q} dQ\right] + \varphi_1 \left[\int_Z^{Q_0} \frac{Z}{2}\frac{t_1}{T}\theta_1 e^{-\theta_1 Q} dQ + \int_{Q_0}^{\infty} \frac{Z}{2}\frac{t_1}{T} e^{Q_0(\theta_2-\theta_1)}\theta_2 e^{-\theta_2 Q} dQ\right] + \varphi_2 \left[\int_Z^{Q_0} \frac{(Q-Z)}{2}\frac{t_2}{T}\theta_1 e^{-\theta_1 Q} dQ + \int_{Q_0}^{\infty} \frac{(Q-Z)}{2}\frac{t_2}{T} e^{Q_0(\theta_2-\theta_1)}\theta_2 e^{-\theta_2 Q} dQ\right] \quad (14)$$

$$\frac{d\psi(Z)}{dZ} = \varphi_1 \left[\theta_1 Z \int_0^{Q_0} e^{-\theta_1 Q} dQ - \theta_1 \int_0^{Q_0} Q\, e^{-\theta_1 Q} dQ + \theta_2 Z e^{Q_0(\theta_2-\theta_1)} \int_{Q_0}^{Z} e^{-\theta_2 Q} dQ - \theta_2 e^{Q_0(\theta_2-\theta_1)} \int_{Q_0}^{Z} Q e^{-\theta_2 Q} dQ\right] + \varphi_1 \frac{Z^2}{2Q}\left[\int_Z^{Q_0} \theta_1 e^{-\theta_1 Q} dQ + \int_{Q_0}^{\infty} e^{Q_0(\theta_2-\theta_1)}\theta_2 e^{-\theta_2 Q} dQ\right] + \varphi_2 \frac{(Q-Z)^2}{2Q}\left[\theta_1 \int_Z^{Q_0} e^{-\theta_1 Q} dQ + e^{Q_0(\theta_2-\theta_1)}\theta_2 \int_{Q_0}^{\infty} e^{-\theta_2 Q} dQ\right] \quad (15)$$

$$\frac{d\psi(Z)}{dZ} = \varphi_1 \left[\theta_1 Z \left[\frac{e^{-\theta_1 Q}}{-\theta_1}\right]_0^{Q_0} - \theta_1 \left(Q\left[\frac{e^{-\theta_1 Q}}{-\theta_1}\right]_0^{Q_0} - e^{-\theta_1 Q}(1)\right) + \theta_2 Z e^{Q_0(\theta_2-\theta_1)} \left[\frac{e^{-\theta_2 Q}}{-\theta_2}\right]_{Q_0}^{Z} - \theta_2 e^{Q_0(\theta_2-\theta_1)} \left(Q\left[\frac{e^{-\theta_2 Q}}{-\theta_2}\right]_{Q_0}^{Z} - e^{-\theta_2 Q}(1)\right)\right] + \varphi_1 \frac{Z^2}{2Q}\left[\left[e^{-\theta_1 Q}\right]_Z^{Q_0} + e^{Q_0(\theta_2-\theta_1)}\left[-e^{-\theta_2 Q}\right]_{Q_0}^{\infty}\right] + \varphi_2 \frac{(Q-Z)^2}{2Q}\left[\left[-e^{-\theta_1 Q}\right]_Z^{Q_0} + e^{Q_0(\theta_2-\theta_1)}\left[-e^{-\theta_2 Q}\right]_{Q_0}^{\infty}\right] \quad (16)$$

$$\frac{d\psi(Z)}{dZ} = \varphi_1 \left[Z\left[-e^{-\theta_1 Q}\right]_0^{Q_0} - \left(Q\left[-e^{-\theta_1 Q}\right]_0^{Q_0} - e^{-\theta_1 Q}(1)\right) + Z e^{Q_0(\theta_2-\theta_1)}\left[-e^{-\theta_2 Q}\right]_{Q_0}^{Z} - e^{Q_0(\theta_2-\theta_1)}\left(Q\left[e^{-\theta_2 Q}\right]_{Q_0}^{Z} - e^{-\theta_2 Q}(1)\right)\right] + \varphi_1 \frac{Z^2}{2Q}\left[\left[e^{-\theta_1 Q}\right]_Z^{Q_0} + e^{Q_0(\theta_2-\theta_1)}\left[-e^{-\theta_2 Q}\right]_{Q_0}^{\infty}\right] + \varphi_2 \frac{(Q-Z)^2}{2Q}\left[\left[-e^{-\theta_1 Q}\right]_Z^{Q_0} + e^{Q_0(\theta_2-\theta_1)}\left[-e^{-\theta_2 Q}\right]_{Q_0}^{\infty}\right] \quad (17)$$

$$\frac{d\psi(Z)}{dZ} = \varphi_1\left[Z\left(1-e^{-\theta_1 Q_0}\right) - \left(Q\left(1-e^{-\theta_1 Q_0}\right) - e^{-\theta_1 Q}\right) + Z e^{Q_0(\theta_2-\theta_1)}\left(e^{-\theta_2 Q_0} - e^{-\theta_2 Z}\right) - e^{Q_0(\theta_2-\theta_1)}\left(Q\left(e^{-\theta_2 Q_0} - e^{-\theta_2 Z}\right) - e^{-\theta_1 Q}\right)\right] + \varphi_1 \frac{Z^2}{2Q}\left[\left(e^{-\theta_1 Z} - e^{-\theta_1 Q_0}\right) + e^{Q_0(\theta_2-\theta_1)} e^{-\theta_1 Q_0}\right] + \varphi_2 \frac{(Q-Z)^2}{2Q}\left[\left(e^{-\theta_1 Z} - e^{-\theta_1 Q_0}\right) + e^{Q_0(\theta_2-\theta_1)} e^{-\theta_2 Q_0}\right] \quad (18)$$

$$\frac{d\psi(Z)}{dZ} = \varphi_1\left[\left(1-e^{-\theta_1 Q_0}\right)(Z-Q) - Q e^{-\theta_1 Q} + \left(e^{-\theta_2 Q_0} - e^{-\theta_2 Z}\right)\left(Z e^{Q_0(\theta_2-\theta_1)} - e^{Q_0(\theta_2-\theta_1)}\right) - e^{-\theta_2 Q}\right] + \varphi_1 \frac{Z^2}{2Q}\left[\left(e^{-\theta_1 Z}\right)\right] + \varphi_2 \frac{(Q-Z)^2}{2Q}\left[e^{-\theta_1 Z}\right] \quad (19)$$

The following assumption is considered while solving equation 19 that the lead time is zero and single period inventory model will be used with the time horizon considered to as finite. When the supply and the level of inventory are same and all the other cases are considered zero. Since analytical solutions to the problem are difficult to obtain. Equation19 is solved using Mathematica 8.0. Hence,

$$\frac{d\psi(Z)}{dZ} = \psi(\hat{Z}) = Z - \frac{1}{\theta_1} - \frac{Z}{e^{\theta_1 Q_0}} + \frac{1}{\theta_1 e^{\theta_1 Q_0}} + \frac{Q_0}{e^{\theta_1 Q_0}}$$

$$-ZQ_0 - \frac{Q}{e^{\theta_1 Q_0}} - Z^2 + \frac{Q e^{\theta_2 Q_0}}{e^{\theta_1 Q_0} e^{\theta_2 Z}}$$

$$= Z - \frac{1}{\theta_1} - \frac{Z}{e^{\theta_1 Q_0}} + \frac{1}{\theta_1 e^{\theta_1 Q_0}} + \frac{Q_0}{e^{\theta_1 Q_0}} \quad (20)$$

Which is the required optimal solution.

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