Novelty Journals

ISSN 2394-9716

International Journal of Novel Research in Interdisciplinary Studies
Vol. 9, Issue 5, pp: (1-6), Month: September – October 2022, Available at: www.noveltyjournals.com

Linear Systems of Fractional Differential Equations

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DOI: https://doi.org/10.5281/zenodo.7140386

Published Date: 03-October-2022

***Abstract*: In this paper, based on Jumarie's modified Riemann-Liouville (R-L) fractional derivative, the linear systems of fractional differential equations is studied. We provide some examples to illustrate how to solve linear systems of fractional differential equations according to the characteristic polynomials. This problem is discussed in three cases: all eigenvalues are real and different, having the same real eigenvalues, and having complex eigenvalues. A new multiplication of fractional analytic functions plays an important role in this paper. In fact, the new multiplication is a generalization of the multiplication of traditional analytic functions.**

***Keywords*: Jumarie's modified R-L fractional derivative, Linear systems of fractional differential equations, Characteristic polynomials, Eigenvalues, New multiplication, Fractional analytic functions.**

I. INTRODUCTION

The history of fractional calculus is almost as long as the development of ordinary calculus theory. In the past few decades, fractional calculus has been applied to many fields, such as physics, dynamics, signal processing, electrical engineering, viscoelasticity, economics, biology, control theory, electronics, etc [1-7]. However, the definition of fractional derivative is not unique. Common definitions include Riemann-Liouville (R-L) type of fractional derivative, Caputo type of fractional derivative, Grunwald-Letnikov (G-L) type of fractional derivative, and Jumarie's modified R-L fractional derivative [8-11]. Since the Jumarie type of R-L fractional derivative makes the derivative of constant function equal to zero, it is easier to use this definition to connect fractional calculus with traditional calculus.

In this paper, based on Jumarie type of R-L fractional calculus, the linear systems of fractional differential equations is studied. We use examples to illustrate how to solve linear systems of fractional differential equations according to the characteristic polynomials. We discuss this problem in three cases: all eigenvalues are real and distinct, having the same real eigenvalues, and having complex eigenvalues. A new multiplication of fractional analytic functions plays an important role in this article. In fact, the new multiplication is a generalization of the multiplication of ordinary analytic functions.

II. DEFINITIONS AND PROPERTIES

Firstly, we introduce the fractional calculus and some properties used in this paper.

Definition 2.1 ([12])**:** Let $0 < \alpha \leq 1$, and t_0 be a real number. The Jumarie's modified R-L α-fractional derivative is defined by

$$\left({}_{t_0}D_t^{\alpha}\right)[f(t)] = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_{t_0}^{t}\frac{f(x)-f(t_0)}{(t-x)^{\alpha}}dx, \quad (1)$$

where $\Gamma(\ )$ is the gamma function.

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Vol. 9, Issue 5, pp: (1-6), Month: September – October 2022, Available at: www.noveltyjournals.com

Proposition 2.2 ([13])**:** *Let* α, β, t_0, C *be real numbers and* $\beta \geq \alpha > 0$, *then*

$$\left({}_{t_0}D_t^{\alpha}\right)\left[(t-t_0)^{\beta}\right] = \frac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)}(t-t_0)^{\beta-\alpha}, \tag{2}$$

and

$$\left({}_{t_0}D_t^{\alpha}\right)[C] = 0. \tag{3}$$

In the following, the definition of fractional analytic function is introduced.

Definition 2.3 ([14])**:** Assume that t, t_0, and a_k are real numbers for all k, $t_0 \in (a,b)$, and $0 < \alpha \leq 1$. If the function $f_\alpha: [a,b] \to R$ can be expressed as an α-fractional power series, that is, $f_\alpha(t^\alpha) = \sum_{k=0}^{\infty} \frac{a_k}{\Gamma(k\alpha+1)}(t-t_0)^{k\alpha}$ on some open interval containing t_0, then we say that $f_\alpha(t^\alpha)$ is α-fractional analytic at x_0. In addition, if $f_\alpha: [a,b] \to R$ is continuous on closed interval $[a,b]$ and it is α-fractional analytic at every point in open interval (a,b), then f_α is called an α-fractional analytic function on $[a,b]$.

Next, we introduce a new multiplication of fractional analytic functions.

Definition 2.4 ([15])**:** Let $0 < \alpha \leq 1$, and t_0 be a real number. If $f_\alpha(t^\alpha)$ and $g_\alpha(t^\alpha)$ are two α-fractional analytic functions defined on an interval containing t_0 ,

$$f_\alpha(t^\alpha) = \sum_{k=0}^{\infty} \frac{a_k}{\Gamma(k\alpha+1)}(t-t_0)^{k\alpha} = \sum_{k=0}^{\infty} \frac{a_k}{k!}\left(\frac{1}{\Gamma(\alpha+1)}(t-t_0)^{\alpha}\right)^{\otimes k}, \tag{4}$$

$$g_\alpha(t^\alpha) = \sum_{k=0}^{\infty} \frac{b_k}{\Gamma(k\alpha+1)}(t-t_0)^{k\alpha} = \sum_{k=0}^{\infty} \frac{b_k}{k!}\left(\frac{1}{\Gamma(\alpha+1)}(t-t_0)^{\alpha}\right)^{\otimes k}. \tag{5}$$

Then

$$\begin{aligned} & f_\alpha(t^\alpha) \otimes g_\alpha(t^\alpha) \\ &= \sum_{k=0}^{\infty} \frac{a_k}{\Gamma(k\alpha+1)}(t-t_0)^{k\alpha} \otimes \sum_{k=0}^{\infty} \frac{b_k}{\Gamma(k\alpha+1)}(t-t_0)^{k\alpha} \\ &= \sum_{k=0}^{\infty} \frac{1}{\Gamma(k\alpha+1)}\left(\sum_{m=0}^{k}\binom{k}{m} a_{k-m} b_m\right)(t-t_0)^{k\alpha}. \end{aligned} \tag{6}$$

Equivalently,

$$\begin{aligned} & f_\alpha(t^\alpha) \otimes g_\alpha(t^\alpha) \\ &= \sum_{k=0}^{\infty} \frac{a_k}{k!}\left(\frac{1}{\Gamma(\alpha+1)}(t-t_0)^{\alpha}\right)^{\otimes k} \otimes \sum_{k=0}^{\infty} \frac{b_k}{k!}\left(\frac{1}{\Gamma(\alpha+1)}(t-t_0)^{\alpha}\right)^{\otimes k} \\ &= \sum_{k=0}^{\infty} \frac{1}{k!}\left(\sum_{m=0}^{k}\binom{k}{m} a_{k-m} b_m\right)\left(\frac{1}{\Gamma(\alpha+1)}(t-t_0)^{\alpha}\right)^{\otimes k}. \end{aligned} \tag{7}$$

Definition 2.5 ([16])**:** Let n be a positive integer, $\left(f_\alpha(x^\alpha)\right)^{\otimes n} = f_\alpha(x^\alpha) \otimes \cdots \otimes f_\alpha(x^\alpha)$ is called the nth power of $f_\alpha(x^\alpha)$.

In the following, we introduce the fractional exponential function and fractional cosine and sine function.

Definition 2.6: The α-fractional exponential function is defined by

$$E_\alpha(t^\alpha) = \sum_{k=0}^{\infty} \frac{t^{k\alpha}}{\Gamma(k\alpha+1)} = \sum_{k=0}^{\infty} \frac{1}{k!}\left(\frac{1}{\Gamma(\alpha+1)}t^{\alpha}\right)^{\otimes k}. \tag{8}$$

Where $0 < \alpha \leq 1$, and t is a real variable. On the other hand, the α-fractional cosine and sine function are defined as follows:

, (9) $$cos_\alpha(t^\alpha) = \sum_{k=0}^{\infty} \frac{(-1)^k t^{2k\alpha}}{\Gamma(2k\alpha+1)} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}\left(\frac{1}{\Gamma(\alpha+1)}t^{\alpha}\right)^{\otimes 2k}$$

and

$$sin_\alpha(t^\alpha) = \sum_{k=0}^{\infty} \frac{(-1)^k t^{(2k+1)\alpha}}{\Gamma((2k+1)\alpha+1)} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \left(\frac{1}{\Gamma(\alpha+1)} t^\alpha\right)^{\otimes(2k+1)} \quad (10)$$

Proposition 2.7: *If* $0 < \alpha \le 1$ *and* c, d *are two real numbers. Then*

$$E_\alpha((c+id)t^\alpha) = E_\alpha(ct^\alpha) \otimes \left(cos_\alpha(dt^\alpha) + isin_\alpha(dt^\alpha)\right). \quad (11)$$

III. EXAMPLES

In this section, we use examples to illustrate how to solve linear systems of fractional differential equations.

Example 3.1: Let $0 < \alpha \le 1$. Find the general solution of the linear system of fractional differential equations

$$\left({}_{t_0}D_t^\alpha\right) \begin{bmatrix} x_\alpha(t^\alpha) \\ y_\alpha(t^\alpha) \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ -2 & -5 \end{bmatrix} \begin{bmatrix} x_\alpha(t^\alpha) \\ y_\alpha(t^\alpha) \end{bmatrix} \quad (12)$$

Solution Since the characteristic equation is

$$\begin{vmatrix} 2-\lambda & 6 \\ -2 & -5-\lambda \end{vmatrix} = 0. \quad (13)$$

It follows that the eigenvalues of $\begin{bmatrix} 2 & 6 \\ -2 & -5 \end{bmatrix}$ are

$$\lambda_1 = -1, \lambda_2 = -2. \quad (14)$$

(i) $\lambda_1 = -1$: $\begin{bmatrix} 2-\lambda_1 & 6 \\ -2 & -5-\lambda_1 \end{bmatrix} v_1 = 0$, then $\begin{bmatrix} 3 & 6 \\ -2 & -4 \end{bmatrix} v_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, and hence the eigenvector $v_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$.

(ii) $\lambda_2 = -2$: $\begin{bmatrix} 2-\lambda_2 & 6 \\ -2 & -5-\lambda_2 \end{bmatrix} v_2 = 0$, then $\begin{bmatrix} 4 & 6 \\ -2 & -3 \end{bmatrix} v_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, and hence the eigenvector $v_2 = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$.

Therefore, the general solution of this linear system of fractional differential equations is

$$\begin{aligned} &\begin{bmatrix} x_\alpha(t^\alpha) \\ y_\alpha(t^\alpha) \end{bmatrix} \\ &= c_1 E_\alpha(-(t-t_0)^\alpha)v_1 + c_2 E_\alpha(-2(t-t_0)^\alpha)v_2 \\ &= c_1 E_\alpha(-(t-t_0)^\alpha)\begin{bmatrix} 2 \\ -1 \end{bmatrix} + c_2 E_\alpha(-2(t-t_0)^\alpha)\begin{bmatrix} 3 \\ -2 \end{bmatrix} \\ &= \begin{bmatrix} 2c_1 E_\alpha(-(t-t_0)^\alpha) + 3c_2 E_\alpha(-2(t-t_0)^\alpha) \\ -c_1 E_\alpha(-(t-t_0)^\alpha) - 2c_2 E_\alpha(-2(t-t_0)^\alpha) \end{bmatrix}. \end{aligned} \quad (15)$$

That is,

$$x_\alpha(t^\alpha) = 2c_1 E_\alpha(-(t-t_0)^\alpha) + 3c_2 E_\alpha(-2(t-t_0)^\alpha), \quad (16)$$

$$y_\alpha(t^\alpha) = -c_1 E_\alpha(-(t-t_0)^\alpha) - 2c_2 E_\alpha(-2(t-t_0)^\alpha). \quad (17)$$

Where c_1 and c_2 are constants.

Example 3.2: If $0 < \alpha \le 1$. Solve the initial value problem of the linear system of fractional differential equations

$$\left({}_0D_t^\alpha\right) \begin{bmatrix} x_\alpha(t^\alpha) \\ y_\alpha(t^\alpha) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} x_\alpha(t^\alpha) \\ y_\alpha(t^\alpha) \end{bmatrix}, \quad (18)$$

$$\begin{bmatrix} x_\alpha(0) \\ y_\alpha(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \quad (19)$$

Solution The characteristic polynomial is

$$p(\lambda) = \begin{vmatrix} -\lambda & 1 \\ -1 & -2-\lambda \end{vmatrix} = (\lambda+1)^2. \quad (20)$$

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Therefore, the eigenvalues of $\begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$ are $\lambda_1 = \lambda_2 = \lambda = -1$. (21)

$\left(\begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} - \lambda I\right) v_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, that is, $\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} v_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, then the eigenvector $v_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

$\left(\begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} - \lambda I\right)^2 v_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, that is, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} v_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. We choose $v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, then

$$\left(\begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} - \lambda I\right)^2 v_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \text{ but } \left(\begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} - \lambda I\right) v_2 \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (22)$$

Thus, the general solution of this linear system of fractional differential equations is

$$\begin{bmatrix} x_\alpha(t^\alpha) \\ y_\alpha(t^\alpha) \end{bmatrix}$$

$$= c_1\, E_\alpha(-t^\alpha) v_1 + c_2\, E_\alpha(-t^\alpha)\left[I + \frac{1}{\Gamma(\alpha+1)} t^\alpha \otimes \left(\begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} - \lambda I\right)\right] v_2$$

$$= c_1\, E_\alpha(-t^\alpha)\begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2\, E_\alpha(-t^\alpha)\left[\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{1}{\Gamma(\alpha+1)} t^\alpha \otimes \left(\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}\right)\right]\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= c_1\, E_\alpha(-t^\alpha)\begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2\, E_\alpha(-t^\alpha)\begin{bmatrix} 1 + \frac{1}{\Gamma(\alpha+1)} t^\alpha & \frac{1}{\Gamma(\alpha+1)} t^\alpha \\ -\frac{1}{\Gamma(\alpha+1)} t^\alpha & 1 - \frac{1}{\Gamma(\alpha+1)} t^\alpha \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_1\, E_\alpha(-t^\alpha) \\ -c_1\, E_\alpha(-t^\alpha) \end{bmatrix} + \begin{bmatrix} c_2 \frac{1}{\Gamma(\alpha+1)} t^\alpha \otimes E_\alpha(-t^\alpha) \\ c_2\left(1 - \frac{1}{\Gamma(\alpha+1)} t^\alpha\right) \otimes E_\alpha(-t^\alpha) \end{bmatrix}$$

$$= \begin{bmatrix} c_1\, E_\alpha(-t^\alpha) + c_2 \frac{1}{\Gamma(\alpha+1)} t^\alpha \otimes E_\alpha(-t^\alpha) \\ -c_1\, E_\alpha(-t^\alpha) + c_2\left(1 - \frac{1}{\Gamma(\alpha+1)} t^\alpha\right) \otimes E_\alpha(-t^\alpha) \end{bmatrix}. \quad (23)$$

Hence,

$$x_\alpha(t^\alpha) = c_1\, E_\alpha(-t^\alpha) + c_2\, \frac{1}{\Gamma(\alpha+1)} t^\alpha \otimes E_\alpha(-t^\alpha), \quad (24)$$

$$y_\alpha(t^\alpha) = -c_1\, E_\alpha(-t^\alpha) + c_2\left(1 - \frac{1}{\Gamma(\alpha+1)} t^\alpha\right) \otimes E_\alpha(-t^\alpha). \quad (25)$$

Where c_1 and c_2 are constants.

Since $\begin{bmatrix} x_\alpha(0) \\ y_\alpha(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, it follows that $c_1 = 1$ and $c_2 = 2$. And hence,

$$x_\alpha(t^\alpha) = \left(1 + 2\, \frac{1}{\Gamma(\alpha+1)} t^\alpha\right) \otimes E_\alpha(-t^\alpha), \quad (26)$$

$$y_\alpha(t^\alpha) = \left(1 - 2\, \frac{1}{\Gamma(\alpha+1)} t^\alpha\right) \otimes E_\alpha(-t^\alpha). \quad (27)$$

Example 3.3: Suppose that $0 < \alpha \leq 1$. Solve the initial value problem of the following linear system of fractional differential equations

$$\left({}_0D_t^\alpha\right)\begin{bmatrix} x_\alpha(t^\alpha) \\ y_\alpha(t^\alpha) \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} x_\alpha(t^\alpha) \\ y_\alpha(t^\alpha) \end{bmatrix}, \quad (28)$$

$$\begin{bmatrix} x_\alpha(0) \\ y_\alpha(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}. \quad (29)$$

Solution The characteristic equation is

$$p(\lambda) = \begin{vmatrix} 3-\lambda & -2 \\ 4 & -1-\lambda \end{vmatrix} = \lambda^2 - 2\lambda + 5 = 0. \quad (30)$$

Thus, the eigenvalues of $\begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}$ are $\lambda_1 = 1 + 2i$, $\lambda_2 = 1 - 2i$. (31)

For $\lambda_1 = 1 + 2i$, from the following equation

$$\left(\begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix} - \lambda_1 I\right) v = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \tag{32}$$

we obtain that

$$\begin{bmatrix} 2+2i & -2 \\ 4 & -2-2i \end{bmatrix} v = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{33}$$

We choose

$$v = \begin{bmatrix} 1+i \\ 2 \end{bmatrix}. \tag{34}$$

Therefore,

$$E_\alpha((1+2i)t^\alpha)\begin{bmatrix} 1+i \\ 2 \end{bmatrix} \tag{35}$$

is a complex-valued solution of Eq. (29).

Since

$$\begin{aligned} &E_\alpha((1+2i)t^\alpha)\begin{bmatrix} 1+i \\ 2 \end{bmatrix} \\ &= E_\alpha(t^\alpha) \otimes \left(cos_\alpha(2t^\alpha) + isin_\alpha(2t^\alpha)\right)\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix} + i\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) \\ &= \begin{bmatrix} E_\alpha(t^\alpha)\otimes\left(cos_\alpha(2t^\alpha) - sin_\alpha(2t^\alpha)\right) \\ 2E_\alpha(t^\alpha)\otimes cos_\alpha(2t^\alpha) \end{bmatrix} + i\begin{bmatrix} E_\alpha(t^\alpha)\otimes\left(cos_\alpha(2t^\alpha) + sin_\alpha(2t^\alpha)\right) \\ 2E_\alpha(t^\alpha)\otimes sin_\alpha(2t^\alpha) \end{bmatrix}. \end{aligned} \tag{36}$$

Thus, the general solution of this linear system of fractional differential equations is

$$\begin{bmatrix} x_\alpha(t^\alpha) \\ y_\alpha(t^\alpha) \end{bmatrix} = c_1\begin{bmatrix} E_\alpha(t^\alpha)\otimes\left(cos_\alpha(2t^\alpha) - sin_\alpha(2t^\alpha)\right) \\ 2E_\alpha(t^\alpha)\otimes cos_\alpha(2t^\alpha) \end{bmatrix} + c_2\begin{bmatrix} E_\alpha(t^\alpha)\otimes\left(cos_\alpha(2t^\alpha) + sin_\alpha(2t^\alpha)\right) \\ 2E_\alpha(t^\alpha)\otimes sin_\alpha(2t^\alpha) \end{bmatrix}. \tag{37}$$

Where c_1 and c_2 are real constants.

Since $\begin{bmatrix} x_\alpha(0) \\ y_\alpha(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$, it follows that

$$\begin{cases} c_1 + c_2 = 1 \\ 2c_1 = 5 \end{cases}. \tag{38}$$

And hence,

$$c_1 = \frac{5}{2}, c_2 = -\frac{3}{2}. \tag{39}$$

So,

$$\begin{aligned} &\begin{bmatrix} x_\alpha(t^\alpha) \\ y_\alpha(t^\alpha) \end{bmatrix} \\ &= \frac{5}{2}\begin{bmatrix} E_\alpha(t^\alpha)\otimes\left(cos_\alpha(2t^\alpha) - sin_\alpha(2t^\alpha)\right) \\ 2E_\alpha(t^\alpha)\otimes cos_\alpha(2t^\alpha) \end{bmatrix} - \frac{3}{2}\begin{bmatrix} E_\alpha(t^\alpha)\otimes\left(cos_\alpha(2t^\alpha) + sin_\alpha(2t^\alpha)\right) \\ 2E_\alpha(t^\alpha)\otimes sin_\alpha(2t^\alpha) \end{bmatrix} \\ &= \begin{bmatrix} E_\alpha(t^\alpha)\otimes\left(cos_\alpha(2t^\alpha) - 4sin_\alpha(2t^\alpha)\right) \\ E_\alpha(t^\alpha)\otimes\left(5cos_\alpha(2t^\alpha) - 3\otimes sin_\alpha(2t^\alpha)\right) \end{bmatrix}. \end{aligned} \tag{40}$$

Consequently, the solution of this initial value problem is

$$x_\alpha(t^\alpha) = E_\alpha(t^\alpha)\otimes\left(cos_\alpha(2t^\alpha) - 4sin_\alpha(2t^\alpha)\right), \tag{41}$$

$$y_\alpha(t^\alpha) = E_\alpha(t^\alpha)\otimes\left(5cos_\alpha(2t^\alpha) - 3\otimes sin_\alpha(2t^\alpha)\right). \tag{42}$$

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IV. CONCLUSION

In this paper, we study the linear systems of fractional differential equations based on Jumarie's modified R-L fractional derivative. According to the characteristic polynomial, we discuss this problem in three cases, namely, all distinct real eigenvalues, having the same real eigenvalues, and having complex eigenvalues. A new multiplication of fractional analytic functions plays an important role in this article. In fact, the new multiplication is a natural operation in fractional calculus. In the future, we will use fractional exponential function and fractional sine and cosine functions to study fractional calculus and fractional differential equations.

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