

Modeling a Lantern Using Sphere Connector

^{1*}Kusno, ²I Made Tirta, ³Firdaus Ubaidillah

^{1*,2,3}Department of Mathematics, University of Jember, East Java, Indonesia

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Abstract: Lanterns are widely used to enhance decorative lighting elements in various setting including interiors, hotels, homes, cafes, gardens, rituals, weddings, festivals, photo booths, and advertising. Their diverse geometric forms, provide an interesting basis for mathematical modeling and construction. This paper presents an approach for applying mathematical equations to model and construct lantern with different geometric configuration. First, the proposed equations are used to model the lantern with one vertical axis with single component and multiple components. Then, we present the lantern construction of vertical-horizontal axes and multi-components. In this configuration, the continuity condition, C^0 and C^1 , are determined to ensure smooth transition. The proposed construction methods are then subsequently simulated, to illustrate the resulting lantern shapes visually. The results demonstrate that the proposed equations and construction procedures can be effectively applied to generate lantern model with different geometric configuration.

Keywords: Lantern; Construction; Axes; Multi-components; Continuity; Quartic Bézier curves.

I. INTRODUCTION

Lanterns are widely manufactured across various countries and used as decorative lighting for interiors (hotels, homes, and cafes), gardens, rituals, weddings, festivals, photo booths, and advertising. Production technology now incorporates laser cutting, automated screen printing, and LEDs equipped with remote controls and light sensors. Typically, the lantern surface is crafted from paper, leather, fabric, or plastic. Lantern designs often utilize geometric forms based on standard shapes such as polyhedra, pyramids, prisms, truncated cones, spheres, hyperboloids, or surfaces of revolution [1,7,8].

Several studies on the mathematical modeling and related areas of lanterns have been introduced. Sweeney et al. (2021) present a model of light propagation in photonic lanterns that uses differential equations and neural networks [11]. Subsequently, Hong et al. (2025) introduce formulas to predict the shape, angle, and elastic energy of lantern structures [2]. However, lantern designs that combine geometric shapes have not yet been introduced; existing approaches remain limited to individual standard geometric objects. Hence, this paper discusses a lantern design that employs spherical connectors and surfaces of revolution to combine these standard geometric objects.

This paper is represented in the following steps. We state the equations of geometric objects in both implicit and explicit forms. Then, we evaluate the connection between these objects using the sphere and the surface of revolution. After that, it discusses the lantern design using the sphere connector and the surface of revolution. Finally, we summarize and conclude the results.

II. MATHEMATIC REPRESENTATION OF LANTERN COMPONENTS

This section discusses parametric and implicit equations for curves and surfaces used to design the lantern's component shapes. After that, we evaluate the continuity C^0 and C^1 of the connection between two curves and surfaces. Finally, the results of the studies are simulated by using MAPLE Software.

2.1 Mathematical Equations to Design the Lantern Components

The parametric and implicit equations employed to design the Component shapes of the lantern consist of quadratic surfaces in space (sphere, ellipsoid, hyperboloid, paraboloid, cone, cylinder), surfaces of interpolation, and revolution. It presents as follows [3,6,9].

1). Sphere with radius R centered at (x_1, y_1, z_1) in the interval $0 \leq s \leq 1, 0 \leq t \leq 0.5$:

$$\mathbf{B}(s, t) = \langle R \cos 2\pi s \cdot \sin 2\pi t + x_1, R \sin 2\pi s \cdot \sin 2\pi t + y_1, R \cos 2\pi t + z_1 \rangle. \quad (1)$$

2). Ellipsoid with radii R_1, R_2, R_3 centered at (x_1, y_1, z_1) in the interval $0 \leq s \leq 1, 0 \leq t \leq 0.5$:

$$\mathbf{E}(s, t) = \langle R_1 \cdot \cos 2\pi s \cdot \sin 2\pi t + x_1, R_2 \cdot \sin 2\pi s \cdot \sin 2\pi t + y_1, R_3 \cdot \cos 2\pi t + z_1 \rangle. \quad (2)$$

3). One sheet hyperboloid in the interval $0 \leq s \leq 1$ and $-a \leq t \leq a$:

$$\mathbf{H}(s, t) = \langle R_1(\cos 2\pi s - t \cdot \sin 2\pi s) + x_1, R_2(\sin 2\pi s + t \cdot \cos 2\pi s) + y_1, t + z_1 \rangle. \quad (3)$$

4). Elliptic paraboloid with radii R_1, R_2 in the interval $0 \leq s \leq 1$ and $-a \leq t \leq a$:

$$\mathbf{P}(s, t) = \langle R_1 \cdot t \cdot \cos 2\pi s + x_1, R_2 \cdot t \cdot \sin 2\pi s + y_1, t^2 + z_1 \rangle. \quad (4)$$

5). Cone with radius R in the interval $0 \leq s \leq 1$ and $-a \leq t \leq a$:

$$\mathbf{K}(s, t) = \langle R \cdot t \cdot \cos 2\pi s + x_1, R \cdot t \cdot \sin 2\pi s + y_1, t + z_1 \rangle. \quad (5)$$

6). Cylinder (open) with radius R in the interval $0 \leq s \leq 1$ and $-a \leq t \leq a$:

$$\mathbf{S}(s, t) = \langle R \cdot \cos 2\pi s + x_1, R \cdot \sin 2\pi s + y_1, t \rangle. \quad (6)$$

7). Circular-plane with radius R centered (x_1, y_1, z_1) and height z_1 in the interval $0 \leq s \leq 1$ and $0 \leq t \leq 1$:

$$\mathbf{L}(s, t) = \langle R \cdot t \cdot \cos 2\pi s + x_1, R \cdot t \cdot \sin 2\pi s + y_1, z_1 \rangle. \quad (7)$$

8) Interpolation Surface of two curves $\mathbf{C}_1(t)$ and $\mathbf{C}_2(t)$ can be expressed in the linear combination of the vectors $\mathbf{C}_1(t)$ and $\mathbf{C}_2(t)$ in the interval $0 \leq s, t \leq 1$ as follows

$$\mathbf{I}(s, t) = (1-s) \mathbf{C}_1(t) + s \mathbf{C}_2(t). \quad (8)$$

9) Let the generating curve $\mathbf{C}(t)$ lie in the meridian plane XOZ , and let $C_x(t)$, $C_y(t)$, and $C_z(t)$ denote the scalar components of the curve $\mathbf{C}(t)$. Then, the surface of revolution about the Z -axis generated by the curve $\mathbf{C}(s)$ can be formulated as

$$\mathbf{R}_z(s, t) = \langle C_x(t) \cos 2\pi s, C_x(t) \sin 2\pi s, C_z(t) \rangle \quad (9)$$

and, when rotated about the X or Y -axis, respectively

$$\mathbf{R}_x(s, t) = \langle C_x(t), C_y(t) \sin 2\pi s, C_y(t) \cos 2\pi s \rangle \quad (10)$$

$$\mathbf{R}_y(s, t) = \langle C_z(t) \cos 2\pi s, C_y(t), C_z(t) \sin 2\pi s \rangle. \quad (11)$$

On the other hand, it can state the sphere, ellipsoid, hyperboloid, and paraboloid centered at (x_1, y_1, z_1) in the implicit form, respectively

$$(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 = r^2 \quad (12a)$$

$$\frac{(x - x_1)^2}{a^2} + \frac{(y - y_1)^2}{b^2} + \frac{(z - z_1)^2}{c^2} = 1 \quad (12b)$$

$$\frac{(x - x_1)^2}{a^2} + \frac{(y - y_1)^2}{b^2} - \frac{(z - z_1)^2}{c^2} = 1 \quad (12c)$$

$$\frac{(x - x_1)^2}{a^2} + \frac{(y - y_1)^2}{b^2} - (z - z_1) = 0 \quad (12d)$$

with r radius of sphere, and a, b, c real constants.

2.2 Connection Between Two Surface Pieces of the Lantern Components

Consider two regular parametric curves $\mathbf{P}(s)$ and $\mathbf{Q}(s)$ with the same orientation in the interval $0 \leq s \leq 1$. The curves contact order 0 and 1, i.e., C^0 and C^1 , if they verify^[4,5,6,10]

$$\mathbf{P}(1) = \mathbf{Q}(0) \text{ and } \mathbf{P}'(1) = \lambda \mathbf{Q}'(0) \quad (13)$$

with λ a real constant. On the other hand, two regular parametric surfaces $S_1(s,t)$ and $S_2(s,t)$ with $0 \leq s, t \leq 1$, they have to meet

$$S_1(s,1) = S_2(s,0) = \Gamma(s);$$

$$S_1^s(s,1) = S_2^s(s,0) \text{ and } S_1^t(s,1) = S_2^t(s,0) \quad (14)$$

or their tangent planes of both surfaces are coplanar (coincide) along the common curve $\Gamma(t)$. Based on Equations (13) and (14), it can evaluate the connections between two surfaces that are defined among Equations (1-12) as follows.

Simulation 1

Let a sphere $B(s,t) = \langle R_1 \cos 2\pi s \sin 2\pi t + 0, R_1 \sin 2\pi s \sin 2\pi t + 0, R_1 \cos 2\pi t + z_1 \rangle$ in the interval $0 \leq s \leq 1, 0 \leq t \leq 0.25$ and a paraboloid $P(s,t) = \langle R_2 t \cos 2\pi s + 0, R_2 t \sin 2\pi s + 0, R_2 t^2 \rangle$ in the interval $0 \leq s \leq 1, 3 \leq t \leq 0$. The surfaces are connected C^0 along the curve $\Gamma(s) = B(s,0.25) = P(s,3)$, if they have to fulfill $R_1 \cos 2\pi s = R_2 \cdot 3 \cos 2\pi s$; $R_1 \sin 2\pi s = R_2 \cdot 3 \sin 2\pi s$; $z_1 = t^2$ or $R_1 = 3 R_2$ and $z_1 = t^2$. Determine the data $R_1 = 6, R_2 = 3$ and $z_1 = 9$, they result Figure 1a. The first derivative $B^s(s,0.25) = P^s(s,3)$ and $B^t(s,0.25) \neq P^t(s,3)$, therefore this connection is not continuous C^1 . In case we respectively select

the surfaces of Equations (12b) and (12c) in the implicit equation $\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{25} = 1$ with $-6 \leq x, y \leq 6, 0 \leq z \leq 6$ and

$\frac{x^2}{9} + \frac{y^2}{16} - \frac{z^2}{25} = 1$ with $-6 \leq x, y \leq 6, -12 \leq z \leq 0$, we find the connection of the two surfaces in Figure 1b.

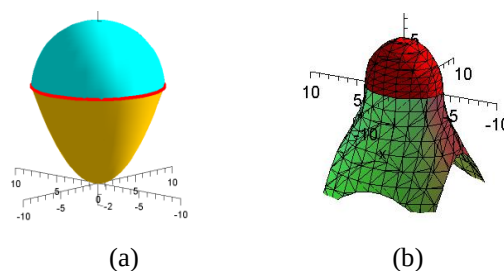


Figure 1. Connection between two surfaces

Let two regular quartic Bézier curves be represented in the form

$$P(t) = P_0(1-t)^4 + 4 P_1(1-t)^3 t + 6 P_2(1-t)^2 t^2 + 4 P_3(1-t) t^3 + P_4 t^4 \quad (15a)$$

$$Q(t) = Q_0(1-t)^4 + 4 Q_1(1-t)^3 t + 6 Q_2(1-t)^2 t^2 + 4 Q_3(1-t) t^3 + Q_4 t^4 \quad (15a)$$

with t in the interval $0 \leq t \leq 1$, and $P_0, P_1, P_2, P_3, P_4, Q_0, Q_1, Q_2, Q_3, Q_4$ control points of the Bézier curves. The first derivatives of the curves are, respectively

$$P'(t) = 4[(P_1-P_0)(1-t)^3 + 3(P_2-P_1)(1-t)^2 t + 3(P_3-P_2)(1-t)t^2 + (P_4-P_3)t^3] \quad (16a)$$

$$Q'(t) = 4[(Q_1-Q_0)(1-t)^3 + 3(Q_2-Q_1)(1-t)^2 t + 3(Q_3-Q_2)(1-t)t^2 + (Q_4-Q_3)t^3]. \quad (16b)$$

From Equation (3), these curves will joint C^0 and C^1 , if they respectively meet $P(1) = Q(0)$ and $P'(1) = \lambda Q'(0)$ that is

$$P_4 = Q_0 \text{ and } (P_4-P_3) = \lambda (Q_1-Q_0). \quad (17)$$

Employing the curves of Equations (15) and (16), we can calculate the surfaces of revolution and their connection defined in Equations (9), (10), and (11) in this way.

Simulation 2

Determining data of control points $P_0 = \langle 8,0,-15 \rangle, P_1 = \langle 6,0,-10 \rangle, P_2 = \langle 2,0,-8 \rangle, P_3 = \langle 6,0,-5 \rangle, P_4 = \langle 4,0,0 \rangle$, and substituting Equation (16a) into Equation (9) presents the surface of revolution in Figure 2a. Determining the control points $Q_0 = P_4 = \langle 4,0,0 \rangle, Q_1 = \langle 3,0,2.5 \rangle, Q_2 = \langle 9,0,7 \rangle, Q_3 = \langle 10,0,12 \rangle, Q_4 = \langle 5,0,15 \rangle$ obtains the connection C^0 of two surfaces of revolution $R_{z1}(s,t)$ and $R_{z2}(s,t)$ that are illustrated in Figure 2b. If we select $Q_1 = \langle 3,0,2.5 \rangle$ such that it verifies $(P_4-P_3) = \lambda (Q_1-Q_0)$ or $\langle -2,0,5 \rangle = 2 \langle -1,0,2.5 \rangle$, then it finds the connection C^1 . In this case, the result of the connection C^1 is smoother than C^0 as shown in Figure 2c.

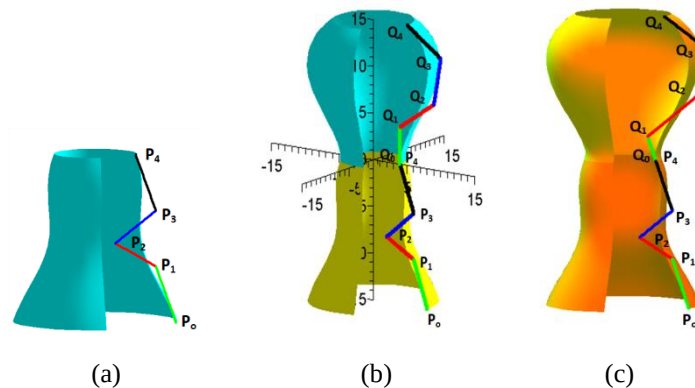


Figure 2. Connection C^0 and C^1 between two surfaces of revolution

III. CONSTRUCTION OF LANTERN

In this lantern construction, we limit the number of axes, the use of mathematical equations in designing its components, and the number of components. The lantern types are designed as follows.

a. Lantern of Single Vertical-axis and One Component

We can design this type with the steps:

1. Determine the Z-axis (vertical axis) to control the symmetrical shape of the lantern component.
2. Select one Equation of Equation (1-12) to design the lantern component and split the parameters s and t of the Equation into some interval series to create various surface shapes of the lantern component.

Simulation 3

Determine ellipsoid of Equation(2) with $R_1 = 4, R_2 = 4, R_3 = 6, x_1 = 0, y_1 = 0, z_1 = 0$, and split the parameters s into some intervals $[0 \leq s \leq 0.05; 0.05 \leq t \leq 0.45], [0.10 \leq s \leq 0.15; 0.05 \leq t \leq 0.45], \dots, [0.90 \leq s \leq 0.95; 0.05 \leq t \leq 0.45]$; and $[0 \leq s \leq 1; 0.05 \leq t \leq 0.1], [0 \leq s \leq 1; 0.40 \leq t \leq 0.45]$. These data result in Figure 3a. Splitting the parameter t into different interval values is illustrated in Figure 3b. Applying a surface of interpolation of Equation (8) with the curves $C_1(t)$ and $[0.8 \times C_2(t)]$ of control points $P_0 = \langle 4, 0, 0 \rangle, P_1 = \langle 8, 0, 4 \rangle, P_2 = \langle 7, 0, 9 \rangle, P_3 = \langle 2, 0, 12 \rangle, P_4 = \langle 9, 0, 15 \rangle$ in the interval $[0 \leq s, t \leq 1]$, and rotating this surface in increment degree per 7.5° yield Figure 3c and Figure 3d.

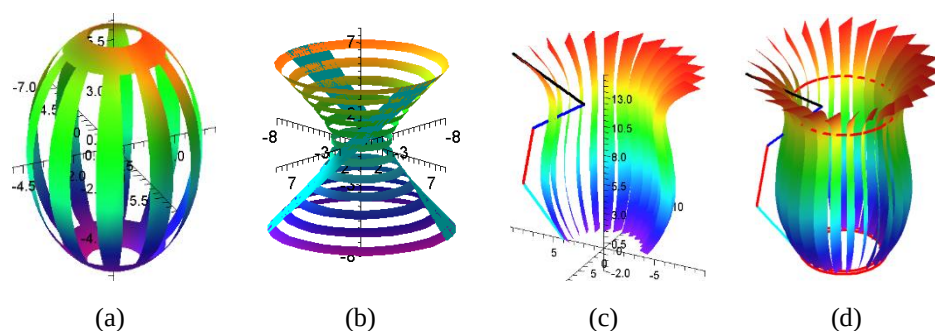


Figure 3. Lantern of Single Vertical-axis and One Component

b. Lantern of One Vertical-axis and Multi-component

Constructing the lantern with one vertical axis and multiple components can be held with the following stages:

1. Determine a vertical line segment that is parallel to the Z-axis (vertical-axis) and split this line into n sub-segments with $2 \leq n \leq 15$.
2. For each sub-segment, select one Equation of Equation (1-12) to construct a lantern component as presented in part (a), and then evaluate the continuity of the connection between two components consecutively by using Equation (13-14).

Simulation 4

We simulate the lantern components shown in Figures 4a and 4b using the following equations. (1) Equation of Cone: $0.5x^2 + 0.5y^2 = (25-z)^2$ with $-3.5 \leq x, y \leq 3.5$ and $23 \leq z \leq 25$; (2) Equation of Cylinder: $x^2 + y^2 = 5$ with $-3 \leq x, y \leq 3$ and $20 \leq z \leq 21.5$; $x^2 + (y-1.6)^2 = 0.05$ with $-1 \leq x, y \leq 1$, $0 \leq y \leq 2$, and $6.8 \leq z \leq 23.7$; $x^2 + (y+1.6)^2 = 0.05$ with $-1 \leq x, y \leq 1$, $-2 \leq y \leq 0$, and $6.8 \leq z \leq 23.7$; $x^2 + y^2 = 2$ with $-3 \leq x, y \leq 3$ and $2.5 \leq z \leq 9$; $x^2 + y^2 = 9$ with $-3 \leq x, y \leq 3$ and $0 \leq z \leq 1$; (3) Equation of hyperboloid: $x^2 + y^2 - (z-10)^2 = 1$ with $-2 \leq x, y \leq 2$ and $9 \leq z \leq 11$; $x^2 + y^2 - (z-8)^2 = 3.4$ with $-4 \leq x, y \leq 4$ and $7 \leq z \leq 10$; (4) Equation of sphere: $\mathbf{B}(s, t) = \langle 3.15 \cos 2\pi s, \sin 2\pi t, 3.15 \sin 2\pi s, \sin 2\pi t, 3.15 \cos 2\pi t \rangle$ with $0 \leq s \leq 1$, $0.05 \leq t \leq 0.2$; (5) Equation of Circular-plane: $\mathbf{L}(s, t) = \langle 4, t \cos 2\pi s, 6, t \sin 2\pi s, 0 \rangle$; $0 \leq s \leq 1$, $0 \leq t \leq 1$; (6) Equation of interpolation surface of two curves $\mathbf{C}_1(t)$ and $\mathbf{C}_2(t)$ in the form $\mathbf{I}(s, t) = (1-s) \mathbf{C}_1(t) + s \mathbf{C}_2(t)$, and then it is rotated in increment degree per 7.5° . In this case, the control points of the curves $\mathbf{C}_1(t)$ are $\mathbf{P}_0 = \langle 2.75, 0, 10 \rangle$, $\mathbf{P}_1 = \langle 4, 0, 11 \rangle$, $\mathbf{P}_2 = \langle 2, 0, 14 \rangle$, $\mathbf{P}_3 = \langle 5.5, 0, 16 \rangle$, $\mathbf{P}_4 = \langle 5, 0, 20 \rangle$ and $\mathbf{C}_2(t) = 0.8 \mathbf{C}_1(t)$.

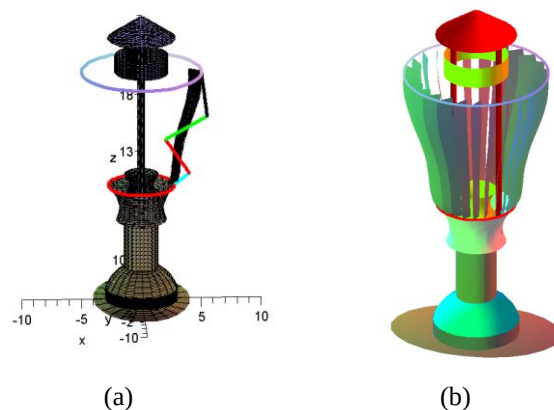


Figure 4. Lantern of One Vertical-axis and Multi-component

c. Lantern of Vertical-horizontal Axes and Multi-component

To design this lantern type, the stages are:

1. Determine $3 \leq m \leq 6$ vertical line segments and $6 \leq n \leq 15$ horizontal line segments, and join these segments such that the constructed lantern can stand up.
2. For each intersection point, define a sphere to connect the lantern components that are designed along the joined segments.
3. Construct the lantern components by using the section (b) method on the joined segments, and then evaluate the continuity of the component connections by using Equation (13-14).

Simulation 5

We illustrate the steps of this method as shown in Figure 5. It needs 8 line-segment equations (Figure 5a) and 16 sphere equations (Figure 5b). Furthermore, we have to define 8 equations for the cylinders, 4 equations for the hyperboloids, and 4 equations for the interpolation surfaces (Figure 5c).

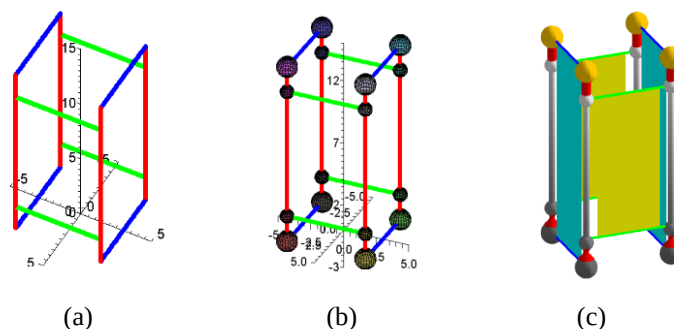


Figure 5. Lantern of Vertical-horizontal Axes and Multi-component

Modeling a single-vertical-axis lantern can be done directly using one of the equations (1–12) that best match the lantern's shape. To achieve a varied surface, the domain interval of the function or equation can first be discretized and subsequently used to formulate the lantern's surface (Figure 3). On the other hand, designing a vertical-axis, multi-component lantern requires selecting several equations (1–12) and subsequently evaluating the continuity of the connections between the lantern components. It can result in various lantern forms with varying volumes at each height level and different visual coloration (Figure 4b). The construction process for a lantern with vertical-horizontal axes and multiple components is generally complex. Among the components are connected by the spheres and the surfaces of interpolation. The result offers a multitude of surfaces and colors, along with a more attractive appearance.

IV. CONCLUSION

It presents some basic equations for designing the components and different shapes of a lantern. They consist of quadratic surfaces in space (sphere, ellipsoid, hyperboloid, paraboloid, cone, cylinder), surfaces of interpolation, and revolution. The results of the modeled lantern features discrete surfaces with a single axis, multi-component surfaces, and varying volumes at each height level. Besides, the designed lantern with vertical-horizontal axes and multiple components offers a multitude of surfaces and colors, resulting in a more attractive appearance. Therefore, these equations and the construction method are expected to be more efficient and useful. After we discuss the lantern's construction oriented by vertical-horizontal axes, it is now important to develop a method for building the lantern using oblique axes.

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