Novelty Journals

ISSN 2394-9651

International Journal of Novel Research in Physics Chemistry & Mathematics
Vol. 10, Issue 1, pp: (1-5), Month: January - April 2023, Available at: www.noveltyjournals.com

SECOND DEGREE HOMOGENEOUS DIOPHANTINE EQUATION WITH THREE UNKNOWNS $x^2+y^2=122z^2$

K HEMA

Assistant Professor, Department of Mathematics, School of Engineering and Technology, Dhanalakshmi Srinivasan University, Trichy,Tamil Nadu, India.

Published Date: 03-January-2023

Abstract: **The homogeneous ternary second degree equation given by $x^2+y^2=122z^2$ is analysed for its non-zero distinct integral points on that. Completely various patterns of the equation into consideration are obtained.**

Keywords: **Ternary, quadratic, Integer solutions, Homogeneous, Diophantine.**

1. INTRODUCTION

It is acknowledge that the quadratic Diophantine equations with 3 unknowns (homogeneous or non-homogeneous) are made in selection[1,2,]. Significantly, one might refer [3-17] for homogeneous or non-homogeneous ternary second degree Diophantine equations that are analysed for getting their corresponding non-zero distinct integer solutions . During this communication, one more attention-grabbing homogeneous ternary quadratic Diophantine equation given by $x^2 + y^2 = 122z^2$ is analysed for its non-zero distinct integer solutions through completely different strategies.

2. METHODS OF ANALYSIS

The ternary second degree equation to be solved for its integer solutions is

$$x^2 + y^2 = 122z^2 \quad (1)$$

Pattern I:

Write 122 as

$$122=(11+i)(11-i) \quad (2)$$

Assume

$$z = a^2 + b^2 \quad (3)$$

Using equations (2), (3) in (1) and using the tactic of resolving, consider,

$$x + iy = (11+i)(a+ib)^2$$

Equating the real and unreal elements, one has

$$x = 11a^2 - 11b^2 - 2ab$$

$$y = a^2 - b^2 + 22ab$$

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Therefore we tend to get,

$$x = 11a^2 - 11b^2 - 2ab$$
$$y = a^2 - b^2 + 22ab$$
$$z = a^2 + b^2$$

Pattern 2:

Equation (1) can also be wriiten as

$$x^2 + y^2 = 121z^2 + z^2$$

$$\Rightarrow x^2 - 121z^2 = z^2 - y^2 \quad (4)$$

(4) can be written within the quantitative relation type as

$$\frac{x+11z}{z+y} = \frac{z-y}{x-11z} = \frac{\alpha}{\beta}, \beta \neq 0 \quad (5)$$

which is adore the system of double equations

$$\beta x - \alpha y + z(11\beta - \alpha) = 0$$
$$\alpha x + \beta y - z(\beta + 11\alpha) = 0$$

Applying the tactic of cross-multiplication to the on-top system of equations, note that

$$x = 11\alpha^2 - 11\beta^2 + 2\alpha\beta$$
$$y = -\alpha^2 + \beta^2 + 22\alpha\beta$$
$$z = \alpha^2 + \beta^2$$

Pattern III :

One can also be written as

$$x^2 + y^2 = 122z^2 * 1 \quad (6)$$

Write 1 as

$$1 = \frac{(3+4i)(3-4i)}{25} \quad (7)$$

Put (3),(7) in (6) and using the tactic of resolving, consider,

$$x + iy = \frac{11+i}{5}(a+ib)^2(3+4i)$$

After Equating the real and unreal terms on either sides , it’s seen that

$$x = \frac{1}{5}\left(29a^2 - 29b^2 - 94ab\right)$$
$$y = \frac{1}{5}\left(47a^2 - 47b^2 + 58ab\right)$$

As our interest is on finding integer solutions replacing a by 5A & b by 5B , we get

$$\left.\begin{aligned} x &= 29A^2 - 29B^2 - 94AB \\ y &= 47A^2 - 47B^2 + 58AB \\ z &= 5A^2 + 5B^2 \end{aligned}\right\} \quad (8)$$

Here (8) and (3) represents non-zero distinct integral solutions of (1).

Pattern IV:

Introduction of the linear transformations

x=u+v,y=u-v,z=2w (9)

in (1) leads to

$$u^2 + v^2 = 244w^2 \quad (10)$$

Assume

$$w = c^2 + d^2 \quad (11)$$

244=(10+12i)(10-12i) (12)

Substituting (11) & (12) in (10) and using the tactic of resolving, consider,

$$u + iv = (10 + 12i)(c + id)^2$$

After Equating the real and unreal terms on either sides , it is seen that

$$\left.\begin{aligned} u &= 10c^2 - 10d^2 - 24cd \\ v &= 12c^2 - 12d^2 + 20cd \end{aligned}\right\} \quad (13)$$

Substituting (13),(11) in (9), we get,

$$\begin{aligned} x &= 22c^2 - 22d^2 - 4cd \\ y &= -2c^2 + 2d^2 - 44cd \\ z &= 2c^2 + 2d^2 \end{aligned}$$

Pattern V:

Equation (10) can also be written as

$$u^2 - 144w^2 = 100w^2 - v^2 \quad (14)$$

Equation (14) can be written within the quantitative relation type as

$$\frac{u + 12w}{10w - v} = \frac{10w + v}{u - 12w} = \frac{\alpha}{\beta},\ \beta \neq 0$$

which is adore to the system of double equations

$$\begin{aligned} \beta u + \alpha v + w(12\beta - 10\alpha) &= 0 \\ \alpha u - \beta v - w(12\alpha + 10\beta) &= 0 \end{aligned}$$

Applying the tactic of cross-multiplication to the on-top system of equations, note that

$$u = -12\alpha^2 + 12\beta^2 - 20\alpha\beta$$
$$v = -10\alpha^2 + 10\beta^2 + 24\alpha\beta$$
$$w = -\alpha^2 - \beta^2$$

Therefore, seeable of (9), the corresponding integer solutions to (1) are given by

$$x = -22\alpha^2 + 22\beta^2 + 4\alpha\beta$$
$$y = -2\alpha^2 + 2\beta^2 - 44\alpha\beta$$
$$z = -2\alpha^2 - 2\beta^2$$

Following the on-top procedure, one might get different set of integer solution to (1).

3. CONCLUSION

In this paper, an endeavour has been created to get non-zero distinct integer solutions to the ternary quadratic Diophantine equation $x^2 + y^2 = 122z^2$ representing homogeneous cone. As there are varieties of cones, the readers might rummage around for alternative varieties of cones to get integer solutions for the corresponding cones.

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