

THEORY OF UNIFORM CIRCULAR MOTION IN PORTFOLIO SELECTION

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Abstract: Scientific forecasting depends on mathematical modelling and statistical modelling. Mathematical models are deterministic, therefore mathematical models are unable to capture the uncertainty in real life. Statistics is known as mathematics of uncertainty. Statistical models contain the randomness; therefore statistical models have become more prominent in forecasting. The theory of Uniform Circular Motion introduced several mathematical models to describe the motion of a particle in a horizontal or vertical circle. These models were extensively used in the fields of Physics and Engineering, but rarely in the fields of Economics, Finance, and Management etc. Yet, in recent past, the theory of Uniform Circular Motion was incorporated with Statistics in forecasting risk and return of Sri Lankan share market. The models developed to measure the return and risk were named as; Circular Model (CM) and Circular Indicator (CI) respectively. Share trading is important to the investors, industries as well as the entire economy of a country. In general, share market investments are high return, but high risk too. A portfolio is a combination of risky assets, which is deemed to minimize the risk and maximize the profit. In modern portfolio theory, total risk of a portfolio is measured by the variance, but the variance is not a suitable measurement for time series data, as they are not independent. This study was focused on combining the CM and CI to develop a Statistical Indicator to give useful information to investors in portfolio selections. Suggested Indicator; named as Coefficient of Stability (CoS) was tested on Sri Lankan share market.

Keywords: Circular model, Circular Indicator, Mathematical Modelling, Statistical Modelling.

I. INTRODUCTION

Newton's theory of uniform circular motion describes the motion of a particle in a circular locus. The theory is widely applied in an Engineering and Physics. The model based on Newton's law is;

$$F = mr\omega^2 \quad (1)$$

Where; m is the mass of the particle in circular motion, r is the radius of the circle, ω is the angular speed and F is the centripetal force.

The model is a mathematical model. Mathematical models are deterministic, not associated with any randomness. This feature is a known weakness of mathematical models, when applying to the real life situations. In contrast, statistical models are associated with randomness. Therefore, statistical models have become more prominent in prediction, control and optimization of the fields of Agriculture, Medicine, Engineering, Military, Humanities and Social Sciences, Economics, Finance and many more.

PROBLEM STATEMENT:

Share trading is an important part of the economy of a country. Share trading affects all the stake holders. Share market investments are exposed to two types of risks; systematic risks and unsystematic risk. Systematic risk cannot be mitigated, but the unsystematic risk can be reduced by diversification. As such, Portfolio investments are preferred than the investments in a single asset. Portfolio selections are based on risk and return of the investment. Hence; forecasting of risk and return were immense interest over the past decades.

The Capital Asset Pricing Model (CAPM) and its improvements, Auto Regressive Integrated Moving Average Models (ARIMA) and Vector Auto Regression (VAR) models were the mainly used models for forecasting share returns. The Circular Model (CM) is a recently developed univariate Statistical model for forecasting share returns. Konarasinghe (2016) has shown that; CAPM and VAR models are not suitable for forecasting individual company returns of the Sri Lankan share market while the ARIMA and CM are capable for the purpose. However, the study has shown that the CM is superior to the ARIMA in Sri Lankan context.

In general, the risk of returns is measured by standard deviation or beta coefficient of Capital Asset Pricing Model (CAPM), but both methods are erroneous (Konarasinghe, 2016). The “Circular Indicator (CI)” is a recently developed statistical indicator for measuring risk of returns.

Konarasinghe (2016) has shown that the CM is a suitable technique for forecasting returns; CI is capable in capturing risk. However, both CM and CI are quite difficult to apply for portfolio selection, due to the advanced mathematical techniques involved in them. Therefore, it is important to develop a mechanism to use those techniques in portfolio selection.

OBJECTIVE OF THE STUDY:

To combine the *CM* and *CI* to make an indicator to help portfolio selection

II. LITERATURE REVIEW

The Circular Model (CM), introduced by Konarasinghe (2016) is given by the formula;

$$R_t = \sum_{k=1}^n (a_k \sin k\omega t + b_k \cos k\omega t) + \varepsilon_t \quad (2)$$

Where; R_t is the return at time t , ω is the angular speed, a_k and b_k are amplitudes, k is the harmonic of oscillation. Development of the CM was based on the Spectral Analysis (Fourier analysis) and the Multiple Regression Analysis. The author has tested the CM on random sample of fifty companies, representing all the business sectors of Colombo Stock Exchange (CSE). Results of the study revealed that the CM was successful in forecasting monthly returns of Sri Lankan share market.

Spectral analysis was initially established in natural sciences such as Physics, Engineering, Geophysics, Oceanography, Atmospheric science, Astronomy etc. and was not much used in the field of Economics. By 1959, John Von Neumann of Princeton University, UK has realized the applicability of Spectral analysis in economic time series. Granger & Morgenstern (1963) was the first recorded application of Spectral analysis in financial markets, but the results of the study were not successful as expected. Granger & Hatanaka (1964) have conducted a study using monthly share prices of the New York stock exchange for the period from January 1946 to December 1960. The results of their study were also not up to the level of satisfaction. It was noted that fewer studies had been conducted on stock returns using Spectral analysis. As Granger and Hatanaka (1964) emphasized, it may be due to the lack of understanding in advanced mathematical techniques: Trigonometry, Calculus and Complex numbers.

The Circular Indicator (CI) also was introduced by Konarasinghe (2016). The development of the Circular indicator was based on model (1) and model (2). Konarasinghe & Abeynayake (2015) has shown that the individual company returns of Sri Lankan stock market follow wave like patterns. Konarasinghe (2016) has combined the idea with the Uniform Circular Motion and developed the Circular Indicator for returns; given by the formula;

$$F_{i,t} = r_{i,t} \cdot \omega_{i,t}^2 \quad (3)$$

Where $F_{i,t}$ is the force on returns of i^{th} company at time t , $r_{i,t}$ is the radius of the circular motion of i^{th} particle at time t and $\omega_{i,t}$ is the angular speed of the circular motion. Accordingly; a company with higher *CI* value is more stable than the company with low *CI* in the market. Konarasinghe (2016) has tested the method on random sample of fifty companies of CSE and concluded that the *CI* is a suitable measurement for risk of returns.

III. METHODOLOGY

Methodology of the study is divided into two parts; Theoretical background and Operationalization of the study.

THEORETICAL BACKGROUND:

The CM was used to estimate the stock returns, the CI was used to measure the risk of returns. Then, an indicator, named “Coefficient of Stability”, was developed to measure the stability of individual companies listed in the share market.

The Circular Model:

The Circular Model (CM) is a univariate forecasting technique, developed to estimate the returns of individual securities. Development of the model was based on the Fourier transformation. Fourier transformation (FT) can be used to transform a real valued function $f(x)$ into series of trigonometric functions (Philippe, 2008). FT has two versions; discrete transformation and continuous transformation. The discrete version of Fourier transformation is;

$$f(x) = \sum_{-\infty}^{\infty} a_n e^{-k\theta} \tag{4}$$

According to De Moivre’s theorem; $e^{-k\theta} = \cos k\theta + i \sin k\theta$ (5)

Where, i is a complex number. Therefore $f(x)$ can be written as:

$$f_x = \sum_{k=1}^n a_k \cos k\theta + b_k \sin k\theta \tag{6}$$

Where a_k and b_k are amplitudes, k is the harmonic of oscillation. The highest harmonic (k) is defined as (Stephen, 1998);

$$k = \begin{cases} n/2: & n \text{ even} \\ (n-1)/2: & n \text{ odd} \end{cases}$$

The Fourier transformation is incorporated to a uniform circular motion of a particle in a horizontal circle and basic trigonometric ratios.

A particle P , which is moving in a horizontal circle of centre O and radius a is given in Figure 1. The ω is the angular speed of the particle;

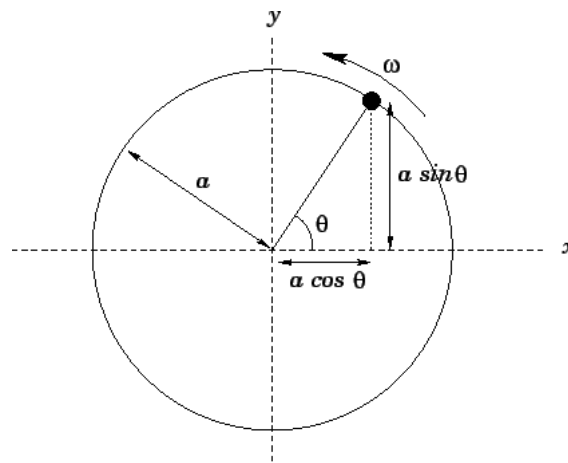


Figure 1: Motion of a particle in a horizontal circle

Angular speed is defined as the rate of change of the angle with respect to time. Then;

$$\omega = \frac{d\theta}{dt}$$

$$\int_0^{\theta} d\theta = \int_0^t \omega dt$$

$$\theta = \omega t$$

Substituting; $f_x = \sum_{k=1}^n a_k \cos k\omega t + b_k \sin k\omega t$ (7)

At one complete circle $\theta=2\pi$ radians. Therefore, the time taken for one complete circle (T) is given by:

$$T = 2\pi / \omega$$
 (8)

Figure 2 and Figure 3 clearly show how to incorporate a particle in horizontal circular motion to trigonometric functions;

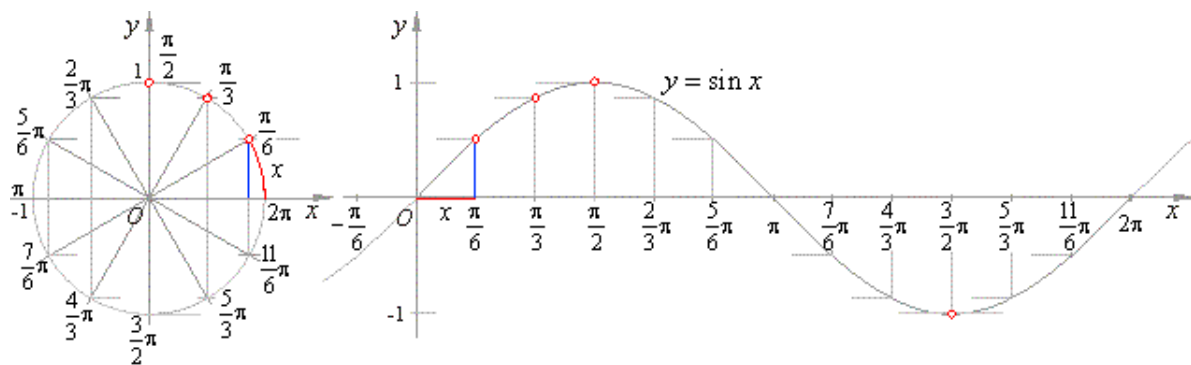


Figure 2: sine function and reference circle

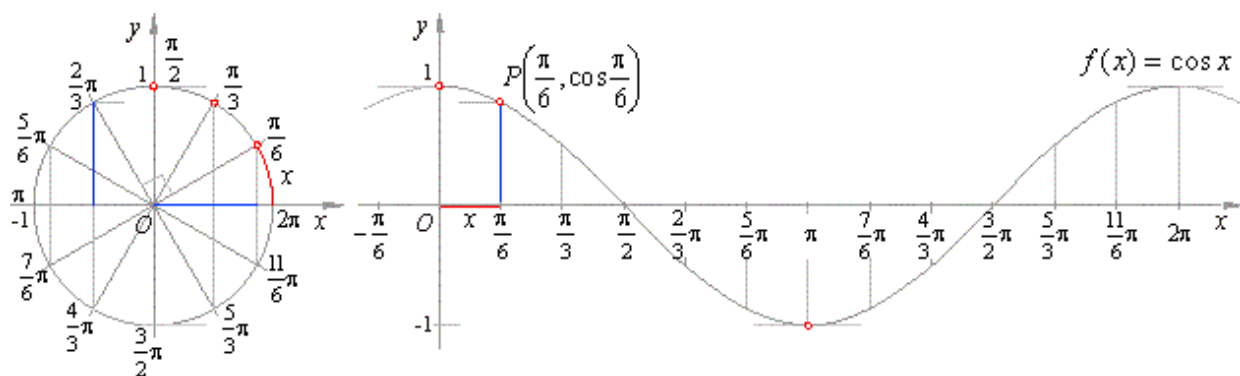


Figure 3: cosine function and reference circle

Reference to Figure 1 ; $\vec{op} = a(\cos\theta i + \sin\theta j)$, where, a is the amplitude or wave height.

A wave with constant amplitude is defined as a regular wave and a wave with variable amplitude is known as an irregular wave.

The concept of Fourier transformation is applied in the present study for explaining returns. In circular motion, the time taken for one complete circle is known as the period of oscillation. In other words, the period of oscillation is equal to the time between two peaks or troughs of sine or cosine function. If a time series follows a wave with f peaks in N observations, its period of oscillation can be given as;

$$T = \frac{\text{total number of periods}}{\text{total number of peaks}} = \frac{N}{f} \quad (9)$$

$$\text{Hence; } \frac{2\pi}{\omega} = \frac{N}{f}$$

$$\omega = 2\pi \frac{f}{N} \quad (10)$$

Hence the return at time t (R_t) was modelled as;

$$R_t = \sum_{k=1}^n (a_k \sin k\omega t + b_k \cos k\omega t) + \varepsilon_t \quad (11)$$

The model (11) was named as the “Circular Model”.

The Circular Indicator:

Development of the circular indicator was based on the uniform circular motion of a particle in a horizontal circle (Newton’s law).

Theory of Uniform Circular Motion of a Particle in a Horizontal Circle:

Reference to Figure 1, position vector of a particle at time t is;

$$\vec{op} = a \cos \theta \mathbf{i} + a \sin \theta \mathbf{j} = a(\cos \theta \mathbf{i} + \sin \theta \mathbf{j})$$

Radius of the circle (a) is a constant; therefore the position vector of the particle at time is a function of θ , but θ vary with time. As such, the magnitude of the velocity or speed of the particle can be obtained by differentiating the position vector of the particle with respect to t and the acceleration of the particle can be obtained by differentiating the velocity vector, with respect to t ;

$$\begin{aligned} v &= \left| \frac{d}{dt} \vec{op} \right| \\ &= \left| \frac{d}{dt} a(\cos \theta \mathbf{i} + \sin \theta \mathbf{j}) \right| \\ &= \left| a(-\sin \theta \mathbf{i} + \cos \theta \mathbf{j}) \frac{d\theta}{dt} \right| \\ &= a \left| -\sin \theta \mathbf{i} + \cos \theta \mathbf{j} \right| \cdot \left| \frac{d\theta}{dt} \right| \\ &= a.1. \left| \frac{d\theta}{dt} \right| \end{aligned}$$

$$\text{Hence; } v = a\omega \quad (12)$$

The acceleration of the particle is obtained by differentiating the velocity. That is

$$\text{Acceleration}(\mathbf{a}) = \frac{d}{dt} \mathbf{v} = \frac{d}{dt} a\omega = a \frac{d^2}{dt^2} \theta$$

$$\text{Hence; } \mathbf{a} = a\omega^2 = \frac{v^2}{a} \quad (13)$$

When the particle moves in a circle, it is constantly changing its direction. At all instances, the particle is moving tangent to the circle. Since the direction of the velocity vector is the same as the direction of the motion, the velocity vector is directed tangent to the circle; as such, the acceleration of the particle also tangent to the circle. Even though the particle is moving under the acceleration with a changing direction, it does not leave the circular path. Therefore, there should be force acting towards the centre of the circle which prevents particle leaving its locus. This force is named as the centripetal force (Hooker, Jennings, Littlewood, Moran and Pateman, 2009).

Using the Newton’s second law of motion; $\mathbf{F} = m\mathbf{a}$ towards the centre;

$$\mathbf{F} = m\omega^2 r = m \frac{v^2}{a} \tag{14}$$

The centripetal force (F) is directly proportional to the mass and the square of the velocity, but inversely proportional to the radius of the circle. In other words the stability of a motion of a particle depends on the mass of the particle, its velocity and the radius of the circular motion.

Konarasinghe (2016) has shown that the share returns of a company follow a uniform circular motion. If mass of the particle (per share return) assumed to be 1, then;

$$F_{i,t} = r_{i,t} \cdot \omega_{i,t}^2 \tag{15}$$

Where $F_{i,t}$ is the force making returns to be in a circular motion of a company i at time t , $r_{i,t}$ is the radius of the circular motion of i^{th} particle at time t and $\omega_{i,t}$ is the angular speed of the circular motion at time t . As such, $F_{i,t}$ was taken as the measurement of risk of returns. That is; larger the $F_{i,t}$, lesser the risk of returns of a company in market performances.

Equation (11); $R_t = \sum_{k=1}^n (a_k \sin k\omega t + b_k \cos k\omega t) + \varepsilon_t$

For a fitted model;

$$\begin{aligned} \bar{R}_t &= \sum_{k=1}^n (a_k \sin k\omega t + b_k \cos k\omega t) \\ &= (a_1 \sin \omega t + b_1 \cos \omega t) + \dots + (a_n \sin n\omega t + b_n \cos n\omega t) \end{aligned} \tag{16}$$

According to (16), the motion comprises of several circular motions with radius a_k , and b_k . Hence $r_{i,t}$ was taken as the average of the radii;

$$r_i = \left(\sum_{i=1}^n |a_i| + |b_i| \right) / n \tag{17}$$

The Coefficient of Stability (CoS):

This study introduces the “Coefficient of Stability (CoS)” as;

$$CoS = \left(\frac{F_t}{R_t} \right) \cdot 100 \tag{18}$$

Where; R_t is the return and F_t is the risk of return. CoS expresses the risk as a proportion of return. As explained in Konarasinghe (2016), if the Circular Indicator value (F_t) is high, the stability is high. As such, higher Coefficient of Stability (CoS) indicates the lesser risk in investment.

Population and Sample of the Study

Listed companies of Colombo Stock Exchange (CSE) in year 2015 were the population of the study. Twenty five companies of CSE were selected by simple random sampling.

Monthly average share returns for individual companies were calculated by a standard formula,

$$R_t = \left(\frac{P_t - P_{t-1}}{P_{t-1}} \right) 100 \quad (19)$$

Where; P_t is the share price at time t .

Outlier Adjustment:

Outliers are extremely large or small values outside the overall pattern of a data set. The outlier detection and adjustment are essential in data analysis. Boundaries of outliers are defined in many ways. Following rule is often used in outlier detection (Attwood, Clegg, Dyer and Dyer, 2008).

$$\begin{aligned} L &= Q_1 - 1.5 * IQR \\ U &= Q_3 + 1.5 * IQR \end{aligned} \quad (20)$$

Where Q_1 , Q_3 are the lower quartile and upper quartile respectively, IQR is the inter quartile range, L is the lower boundary and U is the upper boundary. Any data value above U or below L was considered as outliers. Such data points were adjusted by taking moving average of order three, using a computer program written in MATLAB.

Accuracy of the program is based on two assumptions; first three values of the array are not being outliers, three consecutive outliers have not occurred. Outliers were manually adjusted, when one or two of the assumptions were violated.

Outlier adjusted data were tested on the model; $R_t = \sum_{k=1}^n (a_k \sin k\omega t + b_k \cos k\omega t) + \varepsilon_t$. Goodness of fit tests and

measurements of errors were used in the model validation. The goodness of fit of a statistical model describes how well it fits a set of observations. The plots of residuals versus fits, Auto Correlation Functions (ACF) and Partial Autocorrelation Functions (PACF) of residuals and Ljung-Box Q statistics (LBQ) were used to test the independence of residuals. Histogram of residuals, Normal probability plot of residuals and Anderson Darling test were used to test the normality of residuals.

Measurements of Forecasting Errors:

Forecasting is a part of a larger process of planning, controlling and/ or optimization. Forecast is a point estimate, interval estimate or a probability estimate. One of the fundamental assumptions of statistical forecasting methods is that an actual value consists of a forecast plus an error; In other words, "Error = Actual value – Forecast". This error component is known as the residual. A good forecasting model should have a minimum average of absolute error and zero average of error mean because it should over forecast and under forecast approximately the same (Stephen, 1998).

Measuring errors is vital in the forecasting process. The measurements of errors are divided into two parts; the Absolute measures of errors and the Relative measures of errors. Some absolute measures of errors are; Mean Error (ME), Mean Absolute Deviation (MAD), Sum of Squared Errors (SSE), Root Mean Squared Error (RMSE) and Residual Standard Error (RSE). The absolute measures of errors are very much dependent on the scale of measurement of the dependent variable. Also these measures do not allow comparisons of results over time or between time series. The relative measures of errors are capable in avoiding these disadvantages.

Some relative measures of errors are: Percentage Error (PE), Mean Percentage Error (MPE) and Mean Absolute Percentage Error (MAPE). However, relative measures of errors mislead when data values are extremely small (Stephen, 1998). Also relative measures become undefined when data values are equal to zero. As such absolute measures of errors are better to be used in such a situation. Stock returns, defined by formula (19) may contain zero values. Therefore, relative measures of errors were not used in the study.

IV. RESULTS AND DISCUSSION

Monthly average returns of the companies were calculated by using daily closing share prices. Then outlier adjusted data were tested on the Circular Model, using MATLAB. Then returns (R_t) and risk (F_t) were forecasted by best fitting models; hence CoS's were calculated. Best fitting models for the sample of companies and the summary of the data analysis is given in Table 1.

For example, the fitted CM for the company, TAJ is;

$$R_t = -0.61 - 1.21 \sin 4\omega t + 1.54 \sin 5\omega t + 1.51 \cos 4\omega t \tag{21}$$

The fitted model comprises three trigonometric functions; $\sin 4\omega t$, $\sin 5\omega t$ and $\cos 4\omega t$. In other words the motion of returns comprises three circular motions; circle C1 with angular speed $4\omega t$ and radius 1.21, circle C2 with angular speed $5\omega t$ and radius 1.54 and circle C3 with angular speed $4\omega t$ and radius 1.51.

The average amplitude of the wave (r) is the average of the radii of the reference circles;

$$r = (1.21 + 1.54 + 1.51) / 3 = 1.42$$

The risk of returns was calculated by using the formula $F_{i,t} = r_{i,t} \cdot \omega_{i,t}^2$

Hence, $F_t = 1.42 \times 1.41^2 = 2.82$

Therefore CoS for the company TAJ for May 2015 = $\frac{2.8}{0.5} \times 100 = 560$

The Circular Model was well fitted to 22, out of 25 companies of the sample. It means; the CM is successfully forecast the returns of 88% of the companies. The Coefficient of Stability (CoS) for those 22 companies were calculated, given in Table 1.

Table 1: Summary of the Data Analysis

Company	Best Fitting Model	F_t	R_t	CoS (%)
TAJ	$R_t = -0.61 - 1.21 \sin 4\omega t + 1.54 \sin 5\omega t + 1.51 \cos 4\omega t$	2.8	0.5	700
TRAN	$R_t = 0.65 - 1.51 \sin \omega t$	3.3	2.1	157
ABAN	$R_t = 1.92 + 1.78 \cos 5\omega t$	4.6	-0.8	-575
ACME	$R_t = -0.44 - 2.71 \sin 4\omega t$	10.8	1.2	9
KELA	$R_t = 0.13 + 2.81 \sin 3\omega t - 2.39 \cos 5\omega t$	8.2	3.7	221.6
ROCEL	$R_t = 1.44 + 2.46 \sin 3\omega t - 3.99 \cos 5\omega t$	13.7	-0.2	-6850
BLUE	$R_t = -2.32 + 2.54 \cos 3\omega t$	9.3	-1.9	-489.4
BOGAL	$R_t = -0.78 + 2.37 \sin \omega t + 2.34 \cos \omega t + 2.97 \cos 4\omega t$	14.5	0.1	14500
LCEM	Model does not fit			
ALLI	$R_t = 1.49 + 0.69 \cos \omega t$	4.5	1.5	300
ASIA	$R_t = -0.37 + 2.90 \sin 4\omega t$	10.3	-0.3	-3433.3
DFCC	$R_t = 0.31 - 2.03 \cos 5\omega t$	5.3	0.6	883.3
HNB	$R_t = 0.77 - 1.60 \sin 5\omega t - 1.28 \cos 5\omega t$	5.4	2.4	225
LFIN	$R_t = -0.22 + 2.37 \sin 6\omega t + 2.11 \cos \omega t$	6.9	1.8	383.3
LOLC	$R_t = 1.02 + 1.94 \cos 2\omega t$	4.8	0.1	4800
SAMP	Model does not fit			
HASU	$R_t = 1.08 - 2.10 \sin 5\omega t$	6.1	1.1	554.5
CLAND	$R_t = -0.93 - 1.47 \cos 6\omega t$	3.1	0.3	1033

KELS	Model does not fit			
PDL	$R_t = 0.19 - 1.41\sin 5\omega t - 1.48\cos 6\omega t$	3.1	-1.3	-238.5
EAST	$R_t = -0.34 - 4.37\sin 5\omega t$	26.1	0.1	26100
EQIT	$R_t = 0.50 - 2.13\sin \omega t$	5.0	1.1	454.5
BALA	$R_t = 0.07 - 2.56\sin 3\omega t$	10.7	-2.5	-428
BOGA	$R_t = 0.23 - 3.18\cos 3\omega t$	16.3	3.2	509.4
WATA	$R_t = -1.32 - 2.24\cos 6\omega t$	6.3	0.6	1050

Results reveal that the returns of some companies are positive while the others are negative for the tested month, May 2015. Accordingly; the Coefficient of Stability (CoS) of some companies are positive, while the others are negative. The company EAST has a very high CoS (26100), even though the return is relatively less. Conversely, the company ACME has a quite high return, but very low CoS; as such the investments are at high risk.

V. CONCLUSIONS

Share trading plays a vital role in an economy of a country. A healthy stock market is considered as the key of success in share trading. In the Sri Lankan context, about one third of the GDP contribution is from share trading (Wikipedia, 2016). It is a known fact that the share market investments are high return, but high risk. As such, investors are very much concern about the information in the market. Scientific forecasting plays a vital role in share markets in that light.

The Capital Asset Pricing Model (CAPM), Vector Auto Regression models, ARIMA model and Artificial Neural Network were successfully used forecasting returns in stock markets around the world, but none of the above methods were highly successful in Sri Lankan context. However, Konarasinghe (2016) has introduced the Circular Model (CM) and filled the knowledge gap.

In general, risk of a security is measured by the variance of returns (Pande, 2005). If the observations of a data series are independent, then the variance is a suitable measure of dispersion. But time series data are generally auto correlated, as such, the variance may not be appropriate in measuring the risk of returns. Beta coefficient of CAPM is another way of measuring risk, but the literature revealed that the method is erroneous. The Circular Indicator (CI) is the recent development for measuring risk. However, both risk and returns are equally important in portfolio selections. Investor expectations are high returns at low risk. As such it is essential to have a mechanism to combine the risk and return. This study was focused into that, and suggested the Coefficient of Stability (CoS) for the purpose. It was concluded that the CoS is a suitable measurement, which can be used for portfolio selection. It is recommended to test the method for more companies of Sri Lankan share market as well as the other share markets.

REFERENCES

- [1] Attwood, G., Cope, L., Moran, B., Pateman, L., Pledger, K., Staley, G., & Wilkins, G. (2008). Edexcel AS and A Level Modular Mathematics: Further Pure mathematics 1. Pearson Education Limited, England & Wales.
- [2] Granger, C.W.J., Morgenstem, O. (1963). Spectral Analysis of New York Stock Market Prices. Willey Online Library, International Review of Social Sciences, KYKLOS, 16(1), 1-27.
- [3] Granger, C.W.J., Hatanaka, M. (1964). Spectral Analysis of Economic Time Series. This Weeks Citation Classic, Princeton University Press, Princeton, New Jersey, 299.
- [4] Konarasinghe, W.G.S., Abeynayake, N.R., (2015). Fourier Transformation on Model Fitting for Sri Lankan Share Market Returns. Global Journal for Research Analysis, 4(1), 159-161. Available at: <http://theglobaljournals.com/>
- [5] Konarasinghe, W.G.S., (2016). Model Development for Stock Returns. Doctor of Philosophy Thesis, Postgraduate Institute of Agriculture, University of Peradeniya, Sri Lanka
- [6] Pande, I., M., (2005). Financial Management, 9th Edition. Indian Institute of Management, Ahamedabad.
- [7] Philippe, M. (2008). Analysis of Financial Time Series Using Fourier and Wavelet Methods. Social Science Research Network (SSRN). Retrieved from: <http://ssrn.com/abstract=1289420>
- [8] Stephen, A. D. (1998). Forecasting Principles and Applications. Irwin / McGraw-Hill, USA.