ISSN 2394-7357

International Journal of Novel Research in Civil Structural and Earth Sciences
Vol. 9, Issue 2, pp: (8-15), Month: May - August 2022, Available at: www.noveltyjournals.com

Vibration of Semi Rigid Foundation Locating on Elastic Half Space

Mohammad Shahiq Khan[a], Samir El-Naggar[b], Mohamed M. Soliman[c],
Samir Shalaby[d], Saiful Islam[e]

[a]Associate Professor, College of Engineering & IT, Onaizah Colleges, 51911, Saudi Arabia

[b]Emeritus Prof., Zagazig University, Egypt, Presently: College of Engineering & IT, Onaizah Colleges, 51911, Saudi Arabia

[c]Emeritus Prof., Faculty of Engineering, Benha University, 13511, Egypt

[d]Emeritus Prof., Faculty of Petroleum & Mining Eng., Suez Canal University, 41522 Egypt

[e]Lecturer, College of Engineering, King Khalid University, 62529, Saudi Arabia

DOI: https://doi.org/10.5281/zenodo.6556936

Published Date: 17-May-2022

Abstract: **In this research work, the vibration reaction of a massless or massive raft foundation is investigated. The raft foundation is located on a homogeneous linear-elastic half space and faced to external dynamic loads. The elastic half space, signifying the medium of soil, is formulated by a semi analytical method in a complex response. Hence, this work is analyzed in time-domain by employing inversion fast Fourier transformation for impedance coefficients of subgrade soil. The foundation mat has to be discretized into a number of finite elements, which are combined with the corresponding sub regions of the elastic half-space through compatibility and equilibrium conditions. The proposed approach yields results which are in good agreement with those obtained in previous studies. Numerically works are shown to observe the influence of relative stiffness and aspect ratio of the foundation on the raft reaction.**

Keywords: **Dynamic loads, Elastic half space, Foundation, Semi analytical method, Semi rigid foundation, Vibration.**

I. INTRODUCTION

Most treatments of the soil-structure interaction problem deal with formulations and solutions for the vibration reaction of a subgrade faced to prescribed traction or displacements over the contact area. These treatments pertain to conditions whereby the foundation is considered to be either completely flexible or perfectly rigid. From a mathematical point of view, one could envisage problems in which some functional relationships between traction and displacement are prescribed over the contact area. Such a formulation would describe the interactions of the subgrade with a foundation having finite rigidity. The present investigation deals with such a problem, namely, the response of a rectangular base of finite rigidity locating on elastic half-space. Finite element model has been employed in combination with independent frequency springs and dampers, approximating the soil behavior, to obtain solutions in time-domain. The studies of vibrating reaction of rigid pavement 3-dimensional model under the moving load and temperature gradient reported by the number of authors (e.g. Michael and Edward, 1990; Musharraf et al., 1991; Wang et al., 1992; Taheri and Zaman, 1995; Wu and Shen 1996; David and Parsons, 2001; Shoukry et al., 2007; Patil et al., 2013). Within this review, it has found that the pavement response has significantly effected with the vibrant contact between the pavement and moving load. Sawant et al., (2010 and 2011) also reported parallel work about the vibration reaction of rigid pavement system locating on Pasternak soil medium (Selvadurai 1979). Niki et al., (2018) developed the equations of motion for plate and applied to studies the influence of anisotropy on the reaction for different values of moving load. The dynamic response of rectangular plates has evaluated in contact with linear-elastic half-space (Iguchi, 1978; Savidis and Richter, 1979). Further, the dynamic behavior of massive rectangular plates was studied in frictionless contact with elastic half-space (Whittaker and Cristiano, 1982; Kokkinos and Spyrakos, 1991).

The in plane or crust inflexibility of mat foundations can be considered to be markedly big, when matched to the deformability of soils. Therefore, for horizontal and torsion loading most foundations can be considered to be rigid. But, in several practical conditions, the foundation reaction to erect and rocking load cannot be accurately predicated without accounting for the find out of plane rigidity of the mat. Most studies of the dynamic interaction between foundations and the soil are based on the assumptions of a rigid foundation, however, may not always be valid. Few analyses have been carried out in which the effect of the flexibility on the foundation has been investigated. The main aim of this research is developing a numerical technique to determine the dynamic reaction of flexible base resting on the surface of homogenous elastic half space medium and subjected to external dynamic forces. Special attention has been given to the effect of relative rigidity on the dynamic behavior of the foundation.

II. FORMULATIONS OF PLATE-SUBGRADE SYSTEM

The formulations of the plate by finite element method and the subgrade soil (elastic half-space) by numerical continuum method are presented herein. These formulations are brought out under the following assumptions: 1. the displacements are minor, 2. the central of the plate does not undertake in plane deformation, 3. the crosswise shear deformation is zero, 4. both the foundation material and the soil are linear, elastic and isotropic, 5. the contact between the base of the raft and the soil is smooth and unbounded but no vertical separation occurs.

Raft modeling: The main mechanism governing the behavior of a raft foundation is the out of plane bending which is caused by the downward loading applied at the top of the raft and the upward soil reaction underneath. The plate thickness to be analyzed is assumed to be small compared to the other dimensions. The finite element method has been used to describe the behavior of the raft foundation analysis. For the determination of this problem a simple rectangular, flat and thin elements have been selected for the discrete of foundation geometry. At each node of the element has three degrees of freedom, namely two rotations and one transverse deflection. The vertical deflection is denoted by δ, the rotation about the x-axis is denoted by θx and the rotation about the y-axis is denoted by θy. The element has a total of twelve degrees of freedom. The element shape function is given by:

$$\delta = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x^2 + \alpha_5 xy + \alpha_6 y^2 + \alpha_7 x^3 + \alpha_8 x^2 y + \alpha_9 xy^2 + \alpha_{10} y^3 + \alpha_{11} x^3 y + \alpha_{12} xy^3 \quad (1)$$

Where; αi ($i = 1, 2, 3 \ldots\ldots 12$) are constant coefficients.

After discretizing the foundation into a quantity of elements, the nodal points equilibrium equations can be consequent by numerous different methodologies (Zienkieowicz, 1977). The equation of motion for plate in time domain has been given by:

$$[Mp]\,\{v\} + [Kp]\,\{u\} = \{P\} \qquad (2)$$

Where;

[Kp]: Plate stiffness matrix.

[Mp]: Plate consistent mass matrix.

{P}: Vector of load applied on foundation plate.

{u},{v}:Nodal displacements and accelerat-ions, respectively.

The force vector {P} applied to the plate is the net force acting; that is {P} is given by:

$$\{P\} = \{Q\} - \{F\} \qquad (3)$$

Where;

{Q}: Force vector due to the external dynamic load on the plate.

{F}: Vector of reaction force.

Eq. (2) can be more improved in view of Eq. (3) to:

$$[Mp]\{v\} + [Kp]\{u\} = \{Q\}-\{F\} \qquad (4)$$

Soil implementation: For the vibrant stiffness matrix at the crossing point between the foundation and the soil, the numerical continuum method is used. By supposing the soil as a 3-dimensional semi-infinite elastic medium, the integral equation in the frequency domain treated numerically (Mashhour et al., 1995), can be used. Whereas, the subgrade is represented through impedance coefficients that are associated with discrete point on the contact surface. The numerical clarification of the transient dynamic problem in the frequency domain really contains of a sequences of answers to a steady state dynamic problem for a numeral of distinct values of frequency. The final answer is then found by a numerical inversion of the transformed domain solution to the time domain (John, 1989). Therefore, the vibrant stiffness of soil in the time domain is related with the contact pressure and the displacements in the free field condition as:

$$[Ks]\{w\} = \{Fs\} \qquad (5)$$

Where;

[Ks]: Dynamic stiffness matrix in time domain including the effect of stiffness and radiation damping of soil.

{w}: Vector of elastic half-space displacement

{Fs}: Vector of soil reaction.

Coupling of the Finite Element and a semi-analytical methods: The two basic methods currently available for the analysis of foundation-soil systems are the finite element method and the continuum method. As discussed previously, the continuum approach can treat the three dimensional semi-infinite nature of the soil and the finite element method can be used to describe the flexural plate foundation. Through accurately tracing each nodal point of the finite element discretization to match with the center of the under laying sub region of the half-space (Fig. 1), someone can combine the two substructures by applying the suitable compatibility and equilibrium conditions. By supposing full contact between the two substructures at the nodal points, one can write the compatibility and equilibrium equations as:

$$\{w\} = 0 \qquad (6)$$

$$\{F\} = \{Fs\} \qquad (7)$$

After expanding the vector {w} and the dynamic stiffness matrix [Ks], then, Eqs (4) and (5) may be combined to give:

$$[Mp]\{v\} + \{[Kp] + [Ks]\}\{u\} = \{Q\} \qquad (8)$$

Therefore, the actual displacements of the combined system {u} may be obtained from Eq. (8).

III. NUMERICAL RESULTS AND DISCUSSION

The common time domain procedure described in the earlier sections is used here for the solution of the reaction of three-dimensional massless, massive and flexural raft foundations to numerous varieties of external dynamic loads. This section presents only a selection of cases chosen for their relevance to the main research objective, the evaluation of the role of flexibility in dynamic foundation plate-soil system.

Criteria of rigidity: To investigate the substructures, namely base and soil, it is useful to express the relative stiffness between the two substructures through term, λ. The soil characteristics and both geometric and material properties of the foundations should be considered in determining whether one is designing a rigid or flexible foundation. The stiffness ratio, λ is the principal dimensionless parameter, which characterize the plate-subgrade system. This term has been proposed by Whittaker and Christiano (1982). The extreme values of relative stiffness ratio, λ is varying between the limiting cases of $\lambda = 0.0$ (no plate, free field condition) and $\lambda = \alpha$ (perfect rigid plate).

Comparison with published results: To calculate the convergence of the suggested method, examples of vertical response of square foundation-soil system subjected to vertical harmonic load at plate center are considered. The analysis of flexural foundation is performed in the time domain as described earlier in the previous section. For comparison, the response of foundation-soil system of relative stiffness ratio, $\lambda = 0.004$ (flexible) and 3.3 (rigid) are obtained in two stages. In the first stage, the time domain response of the system for vertical harmonic excitation, with time dependency of the type sin ωt is obtained. The second stage corresponds to the evaluation of the response in frequency domain by means of numerical fast Fourier transform. The considered mesh is chosen 8 x 8 elements similar to the case in the reported literature. The plate is induced by vertical harmonic point load at the geometric center of foundation. The displacement response is prescribed at three characteristic points of the foundation in dimensionless parameters against the dimensionless frequencies as shown in Figs. (2), and (3), and (4). From these figures, it can be seen that the curves of the dynamic displacement of three characteristic points which obtained by the present approach compares very well with the result of Whittaker and Christiano (1982), specially in the case of rigid foundation. However, acceptable difference is noticed with the center point of flexible foundation, $\lambda = 0.004$ probably because the published solution did not take into account the rotation displacements of the plate bending.

Examples: Phenomena such as wind loading and machine vibrations transmit external dynamic forces to foundation system. The forces transmitted to the foundation from the superstructure during seismic excitation may also by viewed as external loading. Previously the responses due to these loads have not rigorously accounted for foundation flexibility. In reality, however, some distortion of foundation does occur under realistic external loading, and the solutions presented in this section illustrate the significance of this distortion. The results of the analysis of two cases of foundation-soil systems are demonstrated in the following subsections.

Massless plate foundation: In the dynamic analysis of machine foundation, the harmonic force generally is utilized to simplify the steady-state vibrations due to reciprocating and rotary machines. One case represents a rectangular, L/B= 2.0, and a square foundation having a relative stiffness factor λ of 0.01 (moderate flexible). This foundation is analyzed under the effect of harmonic point load with specific frequency to illustrate the behavior of foundations under such case of loading. Figs. (5) and (6) display the vertical displacement versus time for characteristic points of the foundations. Here, it is clearly shown the variances in magnitude of displacements and phase angles of the four plotted motions.

Massive plate foundation: The objective of this section is to describe the effect of different parameters on the dynamic response of massive plate. Attention is directed to determining the performance of a massive plate in existence of point step loading at the center. When the stiffness of square plate is taken equal to very small value, $\lambda = 0.001$, the result of displacement history at the center point of foundation is depicted in Fig. (7). Referring to this figure it can be noted that the peak displacement takes place in early duration, then the displacement oscillation decays with time.

IV. CONCLUSIONS

The thrust of the investigation has been focused on the dynamic interaction between an elastic flexural plate and underlying soil media. This investigation provides a good insight on the dynamic behavior of flexural plate-soil system, then more difficult problems can be analyzed. Use of substructure method in soil-structure interface has advantages over direct method, thus decreasing the sizes of the problem and mechanically accounting for radiation situation, hence, excluding the necessity of non-reflective boundaries which are generally working in both the finite element and finite difference methods. For rigid massless foundation, the reaction rest on the elastic properties of the soil medium, the frequency of the exciting dynamic disturbance and the aspect ratio of the foundation. Though, the vibration behavior of a flexible footing is moreover affected by the special distribution of the applied forces and by the material properties of the elastic base. However the relative stiffness would be adequate to describe the elastic reaction of the system to a static load, it is not the only criterion to state whether the system is stiff or flexible when the vibrant load is applied. The nature of the applied loading is also a decisive factor in determining the flexural behavior of the foundation. It has been shown through parametric studies that the flexibility of the foundation is an important factor in assessing the dynamic behavior of foundation. Confined deformations accredited to the flexibility of the foundation can surpass, by orders of magnitude, the overall motion of the foundations, which resembles a rigid-like motion. It is apparent that the flexible foundations are less efficient than rigid foundations in radiating energy into the ground.

REFERENCES

[1] David RB, Parsons ID (2001) Three-dimensional finite element analysis in airport pavement design. International Journal Geomechanics 1(3): 273-290, DOI:10.1080/15323640108500157

[2] Iguchi M (1978) Dynamic interaction of soil-structure with elastic rectangular foundation, Proc.5th Japan Earthquake. Eng. Symp. Tokyo, Japan.

[3] John PW (1989) Soil-structure-interaction analysis in time domain, Nuclear Engineering and Design 111 (3): 381-393. https://doi.org/10.1016/0029-5493(89)90249-5

[4] Kokkinos FT, Spyrakos CC (1991) Dynamic analysis of flexible strip foundations in the frequency domain, Computers & Structures 39 (5):473-482. https://doi.org/10.1016/0045-7949(91)90055-Q

[5] Mashhour M, Abdel-Salam S, El-Naggar S (1995) Dynamics of massless rigid foundation resting on elastic half-space, Third Int. Conf. On Recent Advances in Geotech. Earthquake Eng. and Soil Dynamics, St. Louis Missouri (U. S.A) Vol. 2.

[6] Michael RT, Edward CT (1990) Dynamic response of plates to moving loads: finite element method, Computers & Structures 34 (3):509-521. https://doi.org/10.1016/0045-7949(90)90276-8

[7] Musharraf Z, Taheri MR, Alvappillai A (1991) Dynamic response of a thick plate on viscoelastic foundation to moving loads, Int. J. Num. & Anal. Meth. Geomech. 15 (9): 627-647. https://doi.org/10.1002/nag.1610150903

[8] Niki DB, Yuyan C, Jiang Q (2018) Dynamic response of an elastic plate on a cross-anisotropic elastic half-plane to a load moving on its surface, Transportation Geotechnics 14: 98-106. https://doi.org/10.1016/j.trgeo.2017.11.003

[9] Patil VA, Sawant VA, Deb K (2010) Use of finite and infinite elements in static analysis of pavements, Interaction Multiscale Mechanics 3(1): 95–110. https://doi.org/10.12989/imm.2010.3.1.095

[10] Patil VA, Sawant VA, Deb K (2013) 3D Finite-Element Dynamic Analysis of Rigid Pavement Using Infinite Elements, International Journal of Geomechanics, 13(5): 533-544. DOI:10.1061/(ASCE)GM.1943-5622.0000255

[11] Savidis SA, Richter T (1979) Dynamic response of elastic plates on the surface of the half-space, Int. J. Num. & Anal. Meth. Geomech. 3(3): 245-254. DOI:10.1016/0148-9062(80)91044-x

[12] Sawant VA, Deb K, Patil VA (2010) Dynamic pavement-vehicle interaction of rigid pavement resting on two-parameter soil medium, GeoShanghai 2010 international conference, China, ASCE, Reston Paving Materials and Pavement Analysis, VA, 209–214. http://www.asce.org/Product.aspx?id=2147488425&productid=16698

[13] Sawant VA, Patil VA, Deb K (2011) Effect of vehicle-pavement interaction on dynamic response of rigid pavements, Geomechanics and Geoengineering 6(1): 31–39. DOI:10.1080/17486025.2010.521591

[14] Selvadurai APS (1979) Elastic analysis of soil-foundation interaction, Elsevier Scientific Publishing Company, Amsterdam, Netherlands. eBook ISBN: 9780444596284

[15] Shoukry SN, Fahmy M, Prucz J, William G (2007) Validation of 3DFE analysis of rigid pavement dynamic response to moving traffic and nonlinear temperature gradient effects, International Journal of Geomechanics 7(1): 16–24. DOI: 10.1061/(ASCE)1532-3641(2007)7:1(16)

[16] Taheri MR, Zaman M (1995) Effects of a moving aircraft and temperature differential on response of rigid pavements, Comp. Struct. 57(3): 503–511. https://doi.org/10.1016/0045-7949(94)00625-D

[17] Wang S, Schmid G, Chouw N (1992) Geometrical interaction effects on the dynamic responses of soil-structure system, Computers & Structures 44 (1-2): 367-373. https://doi.org/10.1016/0045-7949(92)90256-Y

[18] Whittaker W, Cristiano P (1982) dynamic response of plate on elastic half-space, J. Eng. Mech. Dive. ASCE 108.

[19] Wu CP, Shen PA (1996) Dynamic analysis of concrete pavements subjected to moving loads, Journal of Transportation Engineering 122(5): 367–373. https://doi.org/10.1061/(ASCE)0733-947X(1996)122:5(367)

[20] Zienkiewicz OC (1977) The finite element method, Mc. Graw-Hill Book Comp. U.K.

APPENDICES – A

List of Figures

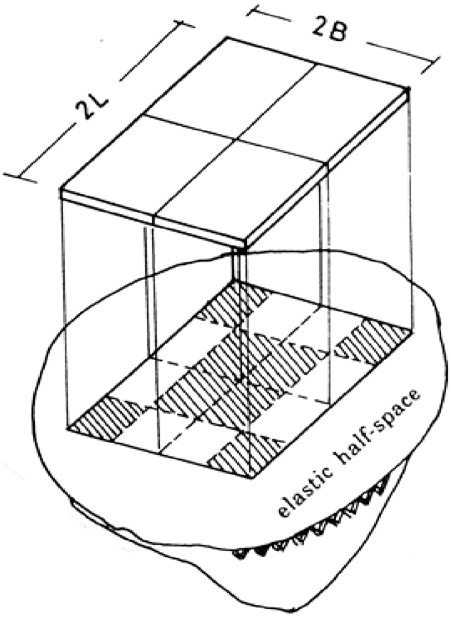

Fig. 1: Plate-soil system

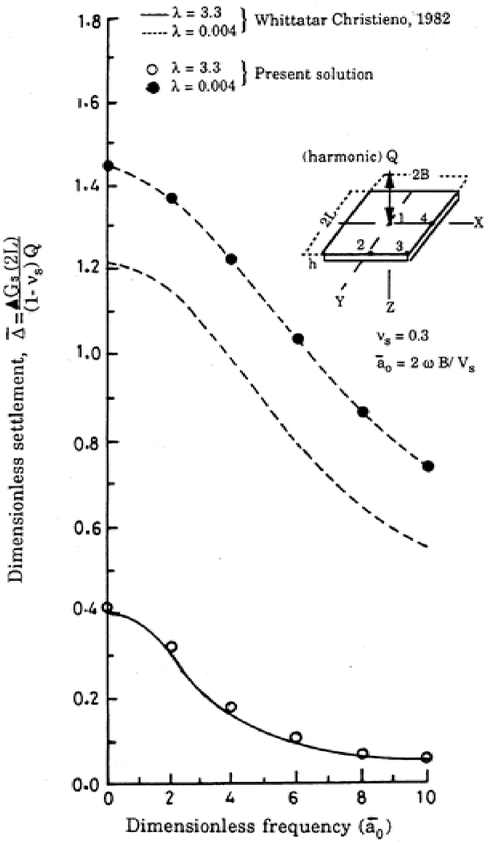

Fig. 2: Harmonic response of a central foundation

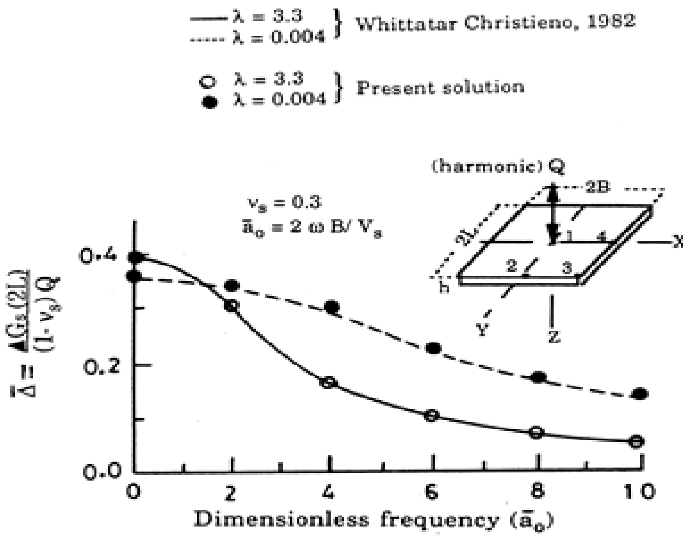

Fig. 3: Harmonic response at edge, point 2

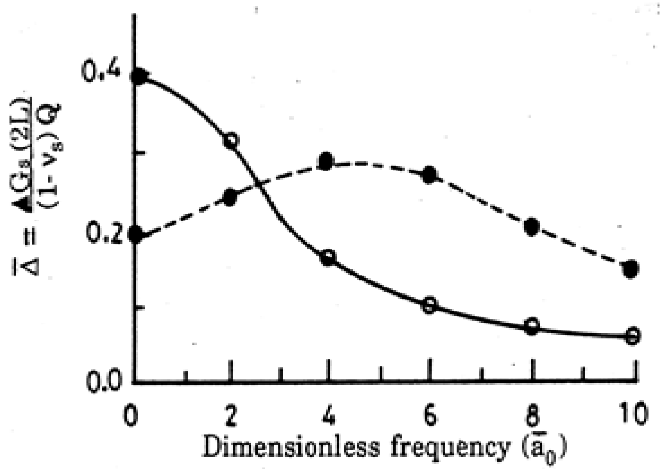

Fig. 4: Harmonic response at corner, point 3

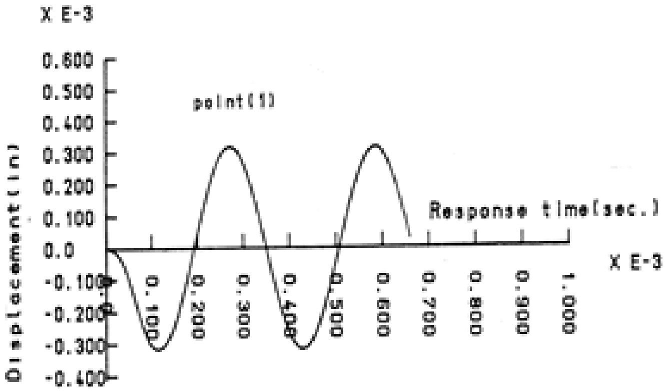

Fig. 5: Displacement history of flexible base at center ((λ= 0.01, L/B = 2.0)

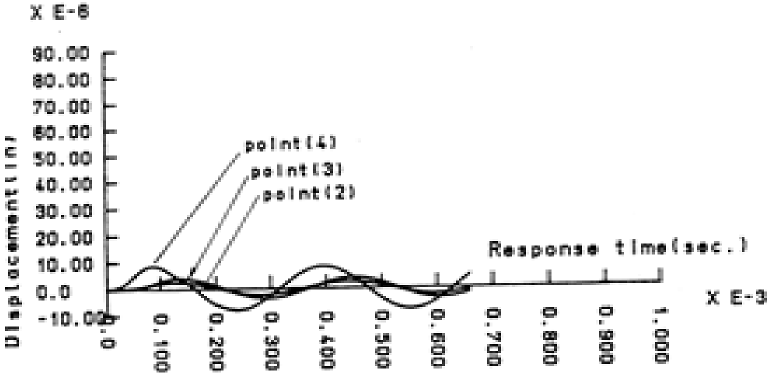

Fig. 6: Displacement history of flexible base at edges (λ= 0.01, L/B= 2.0**)**

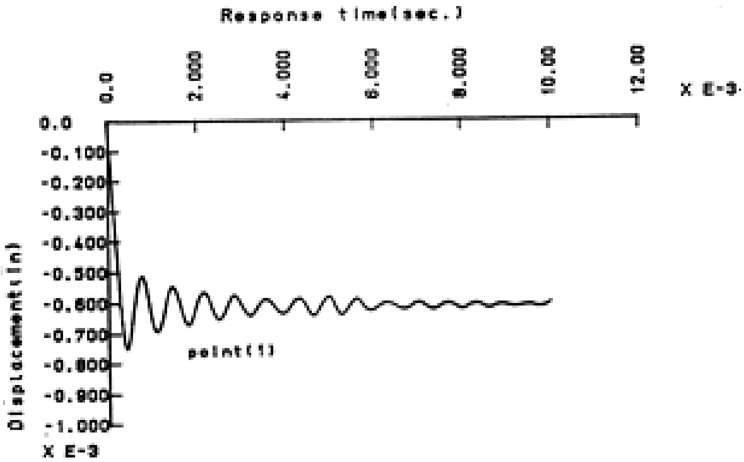

Fig. 7: Displacement history of massive base subjected to point step load ((λ = 0.001, L/B= 1.0**)**