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Volatility Forecasting: An application of a Recurrent Dynamic Neural Networks in Nigeria

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Abstract: **Artificial Neural Networks (ANNs) are non-linear data driven self adaptive approach as opposed to the traditional model based methods. They are powerful tools for modeling, especially when the underlying data relationship is never known. In recent years, Neural Networks have become increasingly popular in finance for tasks such as pattern recognition, classification and time series forecasting. The motivation of this study is to use recurrent dynamic neural networks in enhancing the ability of GARCH models in forecasting the return volatility. A monthly data of Naira/Dollar exchange rate (January, 1991 to January, 2016) and inflation rate (January, 1970 to December, 2015) obtained from central bank of Nigeria's website www.cbn.gov.ng were used in this research. Hybrid models were developed based on the preferred GARCH models and Recurrent Neural Networks to forecast the volatility of these variables. The estimates of the volatilities of the preferred GARCH models were used as inputs and monthly standard deviation as the target of the proposed models. The input to the hybrid I model also included values of other related variables. Simulated series were used as complementary inputs to hybrid model II. The computational results show that GJR-GARCH (1, 2) fits best the Naira/Dollar exchange rate return volatility and EGARCH (2,3) fits best inflation rate returns volatility. The forecasts obtained from these GARCH models and the hybrid models were compared in terms of the standard deviation as a measure of the actual volatility and the results demonstrate that the second hybrid model provides better volatility forecasts.**

Keywords: **volatility, ARCH models, Dynamic recurrent neural networks, Monthly Standard Deviation.**

1. INTRODUCTION

Financial time series plays a vital role in modeling and forecasting volatility of stock markets. The most well-known and classic models include GARCH, EGARCH, and GJR models developed by Bollerslev (1986), (Nelson, 1991) and (Glosten, Jagannathan, & Runkle, 1993) respectively, which cover symmetric and asymmetric effects of news in volatility. These models were extensively applied by researchers in the Nigerian financial Markets (see for example, Musa *et al* (2014), Amaefula and Asare, (2014), Salisu, and Mobolaji, (2013), and Bala and Asemota, (2013)). The assumption of having heteroscedastic errors in time series is suitable with financial market data which are highly volatile (Hajizadeh *et al.*, 2012). While, most of financial data comprise of non-linear dependence pattern, but, is generally assumed to have a linear correlation. Since the volatility is very important for portfolio selection, option pricing and risk management, many researchers have used machine learning techniques, for instance, neural network and support vector machine, to improve the prediction of the financial volatility (Donaldson and Kamstra, 1997). The main reason of using neural network and support vector machine is their flexible abilities to approximate any nonlinear functions arbitrarily without priori assumptions on data distribution (Haykin, 1999). Hence these approaches can cope with the situation that stock market is most of the time heavy tailed and violates normality. In addition, the neural network can capture the stylized characteristics of financial returns such as leptokurtosis, volatility clustering, and leverage effects; hence it generates better prediction of GARCH family (Bildirici and Ersin, 2011).

2. VOLATILITY MODELS

The volatile behavior in financial markets is referred to as the ‘‘volatility’’. Recently, numerous models based on the stochastic volatility process and time series modeling have been found as alternatives to the implied and historical volatility approach. The most widely used model for estimating volatility is ARCH (Auto Regressive Conditional Heteroscedasticity) model developed by Engle in 1982. Since the development of the original ARCH model, a lot of research has been carried out on extensions of this model among which GARCH (Bollerslev, 1986), Generalized Autoregressive Conditional Heteroscedasticity, GARCH (p, q), is a generalization of ARCH model by making the current conditional variance dependent on the p past conditional variances as well as the q past squared innovations. The GARCH (p, q) model can be written as:

$$\varepsilon_t = \sigma_t z_t \qquad (1)$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{P} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \qquad (2)$$

$\omega, \alpha_i, \beta_j$ are nonnegative coefficients, z_t represents a sequence of independent and identically distributed (i.i.d.) random variables with zero mean and unit variance. By definition, ε_t is a serially uncorrelated sequence with zero mean and the conditional variance of σ_t^2 which may be nonstationary. By accounting for the information in the lag(s) of the conditional variance in addition to the lagged ε_{t-i}^2 terms, the GARCH model reduces the number of parameters required. However, ARCH or GARCH models fail to capture asymmetric behavior of the returns. Exponential Generalized Autoregressive Conditional Heteroscedasticity (EGARCH) model was introduced by Nelson (1991) and Nelson and Cao (1992) to account for leverage effects of price change on conditional variance. This means that a large price decline can have a bigger impact on volatility than a large price increase. The EGARCH model can be represented as follows:

$$log\sigma_t^2 = \omega + \sum_{j=1}^{P} \alpha_i \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} - E\left(\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right) \right| + \sum_{j=1}^{q} \beta_j log\sigma_{t-j}^2 + \sum_{k=1}^{r} \gamma_k \left(\frac{\varepsilon_{t-k}}{\sigma_{t-k}}\right) \qquad (3)$$

This model places no restrictions on the parameters α_i and β_j to ensure nonnegative of the conditional variances. In addition to the EGARCH model, another model for capturing the asymmetric features of returns behavior is GJR-GARCH model. The GJR model is closely related to the Threshold GARCH (TGARCH) model proposed by Zakoian (1994) and the Asymmetric GARCH (AGARCH) model of Engle (1990). The GJR model of Glosten et al. (1993) allows the conditional variance to respond differently to past negative and positive innovations. The GJR (p, q) model may be expressed as:

$$\sigma_t^2 = \omega + \sum_{i=1}^{P} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{P} [\gamma_i d(\varepsilon_{t-i} < 0) \varepsilon_{t-i}^2] + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \qquad (4)$$

Where $\gamma_i (i = 1, \ldots, P)$ are the asymmetric parameter and $d(.)$is the indicator function defined such that $d((\varepsilon_{t-i} < 0) = 1$ if $\varepsilon_{t-i} < 0$ and $d((\varepsilon_{t-i} > 0) = 0$ if $\varepsilon_{t-i} > 0$

Similarly, Ding, *et al.* (1993) introduced another asymmetric model APARCH (*p*, *q*) which is written as:

$$\sigma_t^\delta = \omega + \sum_{j=1}^{P} \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^\delta + \sum_{j=1}^{q} \beta_j log\sigma_{t-j}^\delta \qquad (5)$$

where the asymmetric parameter $-1 < \gamma_i < 1 (i = 1 \ldots, P), \delta$ is the non-negative Box-Cox power transformation of the conditional standard deviation process and asymmetric absolute innovations. This power parameter is estimated along with other parameters in the model.

The parameters of GARCH models are estimated using the maximum likelihood method. The log-likelihood function is computed from the product of all conditional densities of the prediction residuals. In this study, we use two penalized model selection criteria, Bayesian information criterion (BIC) and Akaike’s information criterion (AIC), to select best lag parameters for GARCH models (Akaike, 1973; Schwarz, 1998).

3. METHODOLOGY

Neural networks use a non-parametric method of forecasting which means that the underlying non-linear function is not prescribed, ex-ante, explicitly. Thus, the model is not limited to a restrictive list of non-linear functions. In financial applications the most popular class of ANN models has been the single-layer feed forward networks (FNN). In a FNN, information, suitably weighted, is passed from the point of entry (the input layer) to a further layer of hidden neurons. This hidden information is also assigned a weight and finally reaches the output layer which represents the forecast.

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Let $p_t, t = 1,2, \ldots$. be the daily stock index price. The daily returns are then computed by using $r_t = \log(p_t) - \log(p_{t-1})$. The output ,$y$, of a single layer FNN is then given by:

$$y_t = S[a_0 + \sum_{i=1}^{a} a_i \cdot g_{t,i}] \qquad (6)$$

Where $g_{t,i} = G(b_{i0} + \sum_{j=1}^{n} b_{ij} r_{t-j} + \sum_{k=0}^{p} C_{ik} \Delta h_{t-k}^{1/2}), (i = 1, \ldots, q)$ (7)

The inputs to FNN correspond to the returns in the previous n days, following Gençay (1998) and Fernadez-Rodriguez *et.al.* (2000), and the revisions of the estimated conditional volatility,$\Delta h^{1/2}$,over the past p days. As concerns the transfer functions G and S we use the tansig and the purelin function respectively. The tan-sigmoid function normalizes the values of each neuron to be in the interval (-1, +1) while the linear output layer lets the network produce values outside the range -1 to +1. The problem we are faced with is that the FNR has the correct weights such that y has the correct value corresponding to the inputs. This is being accomplished by the *error backpropagation* method under which the neural network runs through all the input data over an initial "training" period and produces a list of outputs. Then the weights are revaluated, by using a recursive "gradient" descent method, so that the mean-squared error between the observed output and the predicted one is minimized. Once the neural network has been trained, it is applied over a different data set covering the so called "validation" period. The purpose here is to evaluate the generalization ability of a supposedly trained network in order to avoid over fitting.

In a dynamic context it is natural to include lagged dependent variables as explanatory variables in the FNN in order to capture dynamics. This problem is being addressed in the relevant literature by constructing recurrent networks, i.e. networks with feedbacks from the hidden neurons, g, to the input layer with delay. The recurrent neural networks (RNN) memorize thus information since its output depends on both current and prior inputs. In this paper we apply the RNN with a single hidden layer and feedback connection from the output of the hidden layer to its input. In a RNN model equation (7) can be re-written as:

$$g_{t,i} = G(b_{i0} + \sum_{j=1}^{n} b_{ij} r_{t-j} + \sum_{k=0}^{p} C_{ik} \Delta h_{t-k}^{1/2} + \sum_{l=1}^{m} \delta_{il} g_{l,t-1}), (i = 1, \ldots, q) \qquad (8)$$

and it is easy to show, with back-substitution, that the output y_t depends on the entire history of the inputs r and $\Delta h^{1/2}$. Such Neural Networks can be illustrated in diagrams as depicted in figure 1.

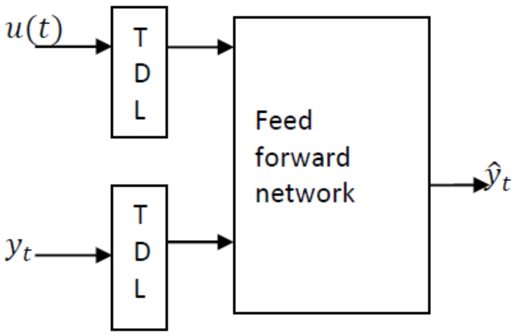

Figure 1: Series Parallel architectures of Recurrent Dynamic neural networks

The trading rule over the testing period works as follows. At the end of each trading day the RNN is being re-estimated over a rolling sample that is equal to the training period set. The output unit, eq. (6), receives the weighted sum of the signals, in the (-1, 1) interval, from eq. (8) and produces a signal through the output transfer function (S). If the value of the signal is greater than zero it is interpreted as a "buy" signal for the next trading day while a value less than zero as a "sell" signal. Then, the total return of the strategy, when transaction costs are not considered, is estimated as:

$$\hat{R}_t = \sum_{t=0}^{N} \hat{y}_t r_t \qquad (9)$$

Where $\hat{y}_t$ is the recommended position which takes the value of (-1) for a short position and (+1) for a long position (see e.g. Fernadez-Rodriguez *et al*., 2000)

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4. HYBRID MODELS

In this study two hybrid models were proposed to forecast volatility of some financial variables in the financial market of Nigeria namely inflation rate and Naira/Dollar Exchange rate. In the construction of the first hybrid model also known as Hybrid model I, the underlying concept was that there are some endogenous variables related to the historical performance of the returns that affect the future price returns volatility in the market such as price returns, squared price returns, price, price squared, etc (based on the preferred GARCH model) together with the volatility estimates were used as input variables to the RNN model and the monthly standard deviation is used to train the network(target). Similarly, the second hybrid model also known as Hybrid model II, in addition to these variables also used the simulated synthetic series from the preferred GARCH model as signals to the network. This is in order to keep the properties of the best fitted GARCH model while enhancing it with RNN model.

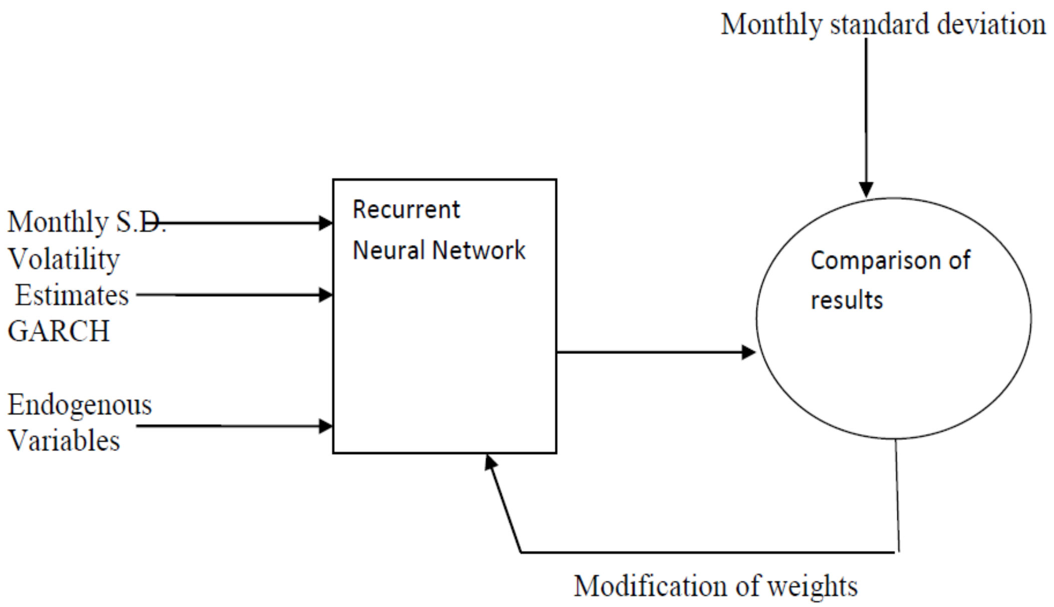

Figure 2: Schematic representation of hybrid model I

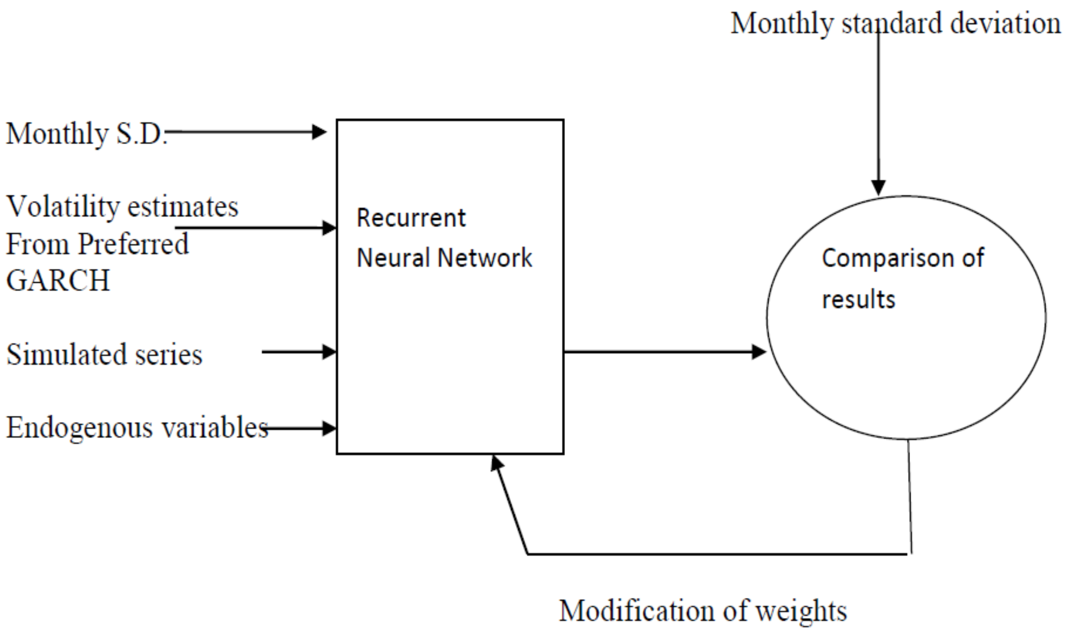

Figure 3: Schematic representation of hybrid model II

5. PERFORMANCE COMPARISON

The performance of the proposed hybrid models in forecasting the volatility was evaluated in comparison with the monthly standard deviation as a measure of the actual volatility. The monthly standard deviation on month t is calculated by

$$\sigma_t = \sqrt{n^{-1} \sum_{i=t}^{t+n} (R_i - \bar{R})^2}, \qquad (10)$$

Where R_i is logarithmic return,$\bar{R} = \sum_{i=1}^{n} {R_i}/{n}$, and Where R_i is logarithmic return,$\bar{R} = \sum_{i=1}^{n} {R_i}/{n}$, and n is the number of months before nearest expiry option.

In addition, three measures are used to evaluate the performance of models in forecasting volatility as follows: root mean square error (RMSE), mean absolute error (MAE), and mean absolute percentage error (MAPE). These measures are defined as:

$$RMSE = \left(n^{-1} \sum_{t=1}^{n} (\sigma_t - \hat{\sigma}_t)^2\right)^{1/2}, \qquad (11)$$

$$MAE = n^{-1} \sum_{i=1}^{n} |\sigma_t - \hat{\sigma}_t|, \qquad (12)$$

$$MAPE = n^{-1} \sum_{i=1}^{n} \left|\frac{\sigma_t - \hat{\sigma}_t}{\hat{\sigma}_t}\right| * 100, \qquad (13)$$

6. DATA CHARACTERISTICS

This study focused on monthly prices of Naira/Dollar Exchange rate and Inflation rate in Nigeria. The exchange rate covers the period of January, 1991 to January, 2016 and inflation rate covers the period of January, 1995 to December, 2015. All data were generated from Central Bank of Nigeria through the website www.cbn.gov.ng. The logarithmic returns were calculated from the data.

Table 1: Data Description and preliminary statistics

Variables	Naira/Dollar Exchange rate returns	Inflation rate returns
Mean	1.094372	1.007706
Standard Deviation	3.911124	1.680736
Skewness	1.364932	0.664962
Kurtosis	10.53829	5.627630
Jarque-Bera[a]	803.4743(0.0000)*	90.70653(0.000)*
$Q^2(15)^b$	35.569(0.001)*	26.692(0.021)*
ARCH test (15)[c]	60.57015(0.000)*	33.80322(0.0036)*
Observation	300	251

[a] Jarque-Bera test of normality

[b] is the Ljung-Box Q test for the 15th order serial correlation of the squared returns

[c] Engle's ARCH test also examines for autocorrelation of the squared returns.

Table 1 indicates that there are significant price fluctuations in the markets as suggested by positive standard deviation. The positive skewness indicates that there is high probability of losses in the market. The excess value of kurtosis suggests that the market is volatile with high probability of extreme occurrences. Moreover, the rejection of Jarque-Bera test of normality shows that the returns deviate from normal distribution significantly and exhibit leptokurtic. The Ljung Box statistic for squared return and Engle ARCH test prove the exhibition of ARCH effects in the returns series.

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Therefore, it is appropriate to apply GARCH, EGARCH and GJR models. Figures 4 and 5 show the monthly exchange rate price and inflation rate with their logarithmic returns. Informally, they suggest that prices are trending or non stationary, the returns concentrate around zero

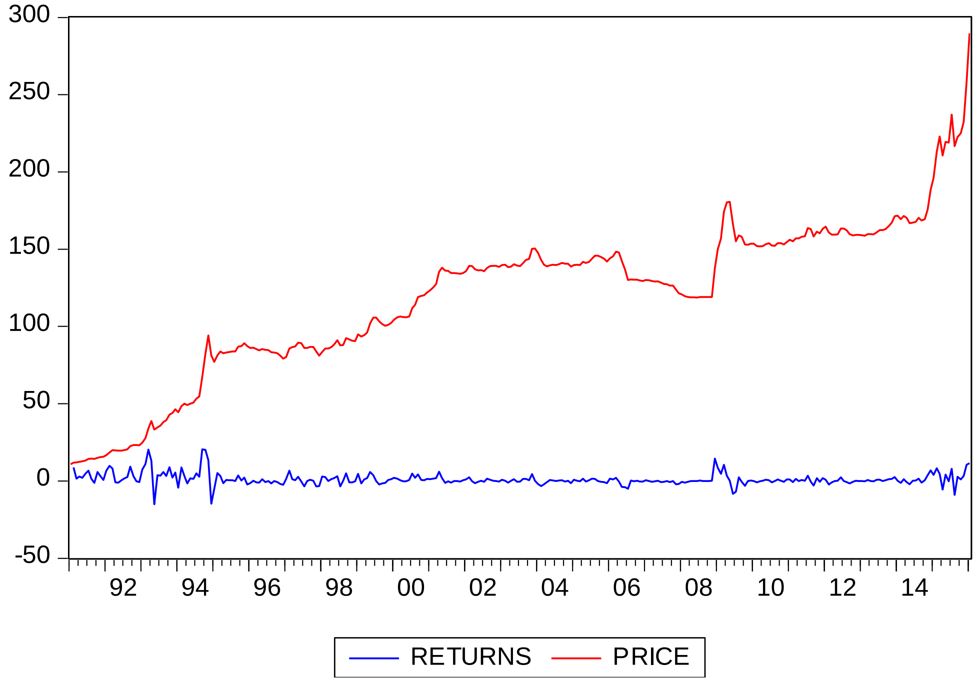

Figure 4: Monthly price and logarithmic returns of Naira/Dollar Exchange

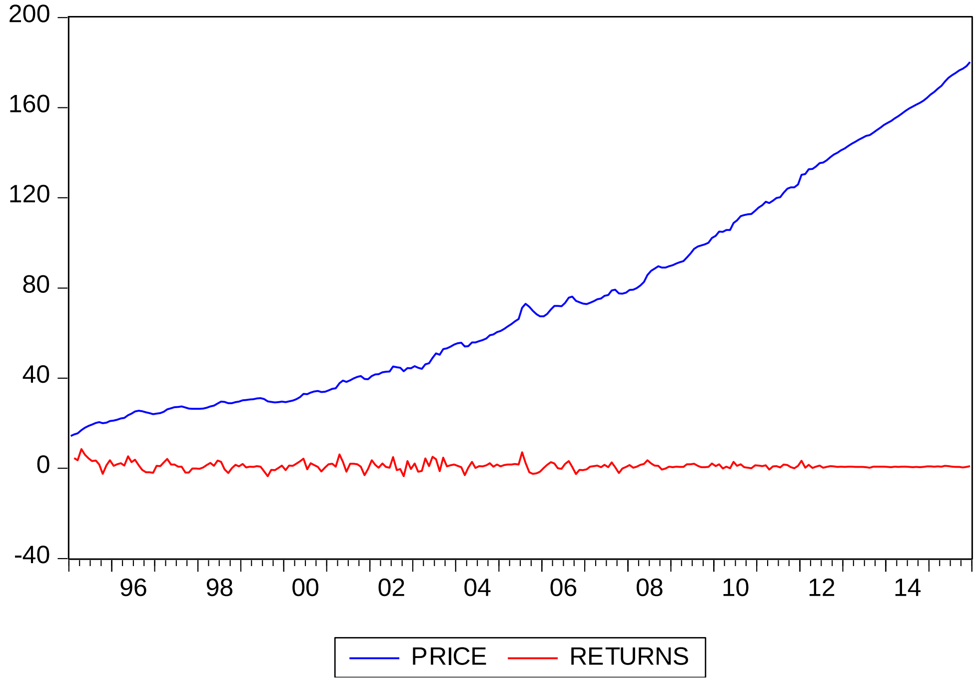

Figure 5: Monthly price and logarithmic returns of inflation rate in Nigeria

7. COMPUTATIONAL RESULTS

In this section, we report on the results of applying GARCH-type models as well as the proposed hybrid models for forecasting volatility of Naira/Dollar Exchange rate and inflation rate returns. As the first step, GARCH, EGARCH and GJR-GARCH models with various combinations of (p, q) parameters ranging from (1, 1) to (3, 3) were calibrated on

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historical returns data. The best models turned out to be GJR (1, 2) according to AIC and BIC criteria for exchange rate return and EGARCH (2, 3) for inflation rate return. Table 2 shows AIC and BIC values for three GARCH-type models with best combinations of (p, q) from each category of GARCH models. To examine the fitness of these models, each of them has been used to forecast the volatilities for 12 months ahead and the results are reported in Tables 3 and 4, respectively.

Table 2: AIC and BIC criteria for the preferred models

Variables	GARCH Models	Selection criteria	
Naira/Dollar Exchange rate log returns	GARCH(1,1)	AIC	4.919005
		BIC	4.990885
	GJR(1,2)	AIC	4.788970
		BIC	4.875610
	EGARCH(1,3)	AIC	4.864597
		BIC	4.963606
Inflation rate log returns	GARCH(3,3)	AIC	3.104795
		BIC	3.231567
	GJR(3,3)	AIC	3.171412
		BIC	3.312271
	EGARCH(2,3)	AIC	3.103530
		BIC	3.230302

Table 3: GARCH models performance to volatility forecasting of Naira/Dollar exchange rate for 12-Months ahead

Performance measures	GARCH(1,1)	GJR(1,2)	EGARCH(1,3)
RMSE	3.931231	3.917121	3.940280
MAE	2.271569	2.264131	2.295048
MAPE	214.7706	185.7439	265.3997

Table 4: GARCH models performance to volatility forecasting of inflation rate for 12-Months ahead

Performance measures	GARCH(3,3)	GJR(3,3)	EGARCH(2,3)
RMSE	1.674910	1.675318	1.663503
MAE	1.119766	1.119293	1.113286
MAPE	120.5513	121.2852	114.4601

According to the values of fitness measures, GJR-GARCH (1, 2) and EGARCH (2, 3) have shown the best performance for forecasting volatilities of Naira/Dollar Exchange rate returns and inflation rate returns respectively and thus selected for construction of hybrid models. The estimated volatilities of these models were obtained and used as the network target.

Figure 6 and 7 are examples of the trained networks for forecasting volatilities of the two financial variables involved.

Tables 5 and 6 present the result of the applications of the proposed hybrid models for forecasting volatilities in 12 months ahead. The computational results show that both hybrid models outperform EGARCH model. Hybrid model II exhibits better ability to forecasting volatility of the real market return with respect to all four fitness measures. That is due to the inclusion of simulated series as extra inputs to hybrid model II.

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Table 5: Hybrid models performance to volatility forecasting of Naira/Dollar exchange rate for 12-Months ahead

Measure	GJR(1,2)	Hybrid model I	Hybrid model II
RMSE	3.917121	1.0246112	0.891262
MAE	2.264131	1.0095420	0.654241
MAPE	185.7439	45.321751	27.48921

Table 6: Hybrid models performance to volatility forecasting of Inflation rate for 12-Months ahead

Measure	EGARCH(2,3)	Hybrid model I	Hybrid model II
RMSE	1.663503	1.003281	0.456821
MAE	1.113286	0.769431	0.452791
MAPE	114.4601	24.12316	23.87591

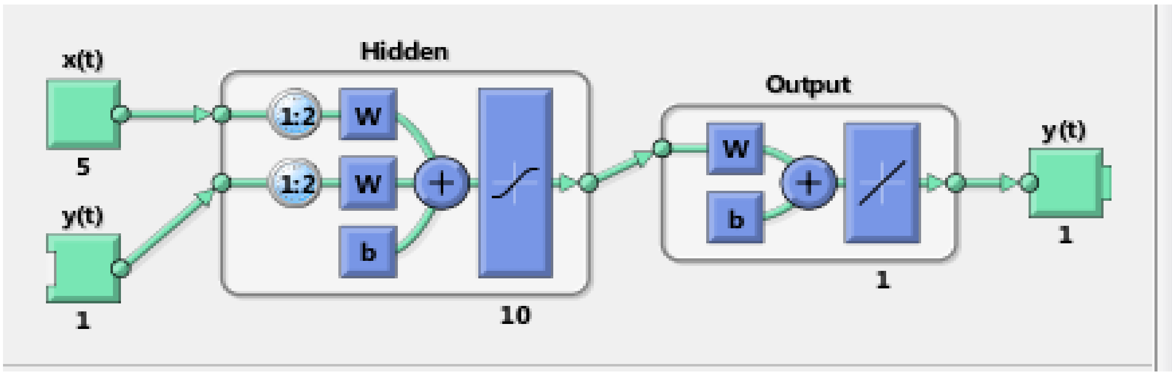

Figure 6: Example of a trained hybrid model I architecture.

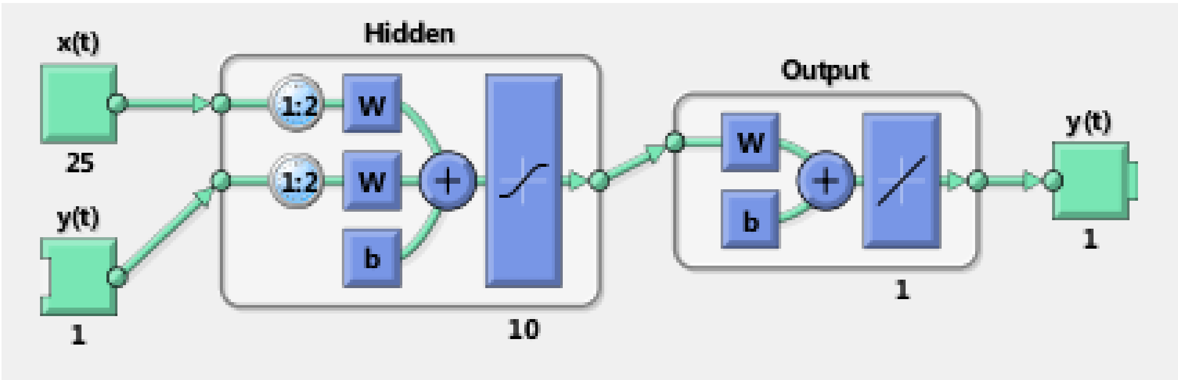

Figure 7: Example of trained hybrid model II architecture with 20 more extra simulated series.

8. CONCLUSION

In this research, we considered using recurrent dynamic neural networks as techniques which can be used to enhance the ability of GARCH models in volatility forecasting. The application of GARCH models in forecasting volatility always gives better results in relatively stable markets, and need to be combined with other models to capture violent and fluctuating markets ((Hajizadeh *et al.*, 2012).

The aim of this research was to forecast the volatility of Naira/Dollar exchange rate and inflation rate returns in Nigeria. Three types of GARCH models were estimated and used for forecasting the volatility of these returns and their performances evaluated using some performances metrics. The best models were GARCH-GJR (1, 2) and EGARCH (2, 3) for Naira/Dollar exchange rate and inflation rate returns respectively. The ability of GJR (1, 2) and EGARCH (2, 3) to forecast was improved by integrating them in to recurrent dynamic neural networks of which two hybrid models were built. The input of the first hybrid model was the volatility estimated by selected GARCH models and some related endogenous variables. Furthermore, the second hybrid model takes simulated volatility series as extra inputs. Such inputs have been intended to characterize the statistical properties of the volatility series when fed into Recurrent Neural Networks. The hybrid models were taught by the monthly standard deviation as a target. Both the hybrid models and the preferred GARCH models were compared in performances in terms of the monthly standard deviation as a measure of the actual volatility. The results finally show that the second hybrid model with the simulated series as extra inputs forecasts volatility better than the other models for both Naira/Dollar exchange rate and inflation rate returns. This model makes better the forecasts of volatility in the Nigerian financial market.

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