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Leveraging ICT to Improve the Understanding of Stereometry among Students in Primary Education

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Abstract: **This paper reports on the design and implementation of an ICT-based learning paradigm referred to the spatial geometry and visual thinking in primary education, based on the 'Model of phases-methods Combinations'. The main scope of this paper is to help 12-year old students of primary education to explore the various types of three-dimensional figures, so as to be capable of manipulating geometrical figures in space, i.e to learn how to compute the surface and the volume of a solid and how to solve complex three-dimensional geometric problems (e.g. arguments related to three-dimensional shapes and geometrical constructs using hard paper or other materials). An ICT-based didactical scenario, that employs a playful and conceivable approach, is presented here, to motivate students of primary education to construct, observe and manipulate geometrical figures in space, and also to guide teachers how to help students construct a successful understanding of stereometry.**

Keywords: **cognitive apprenticeship theory, didactics of geometry, mathematics, model of p-m combinations, primary education, RECOMPP, SFW, stereometry, van Hiele theory.**

1. INTRODUCTION

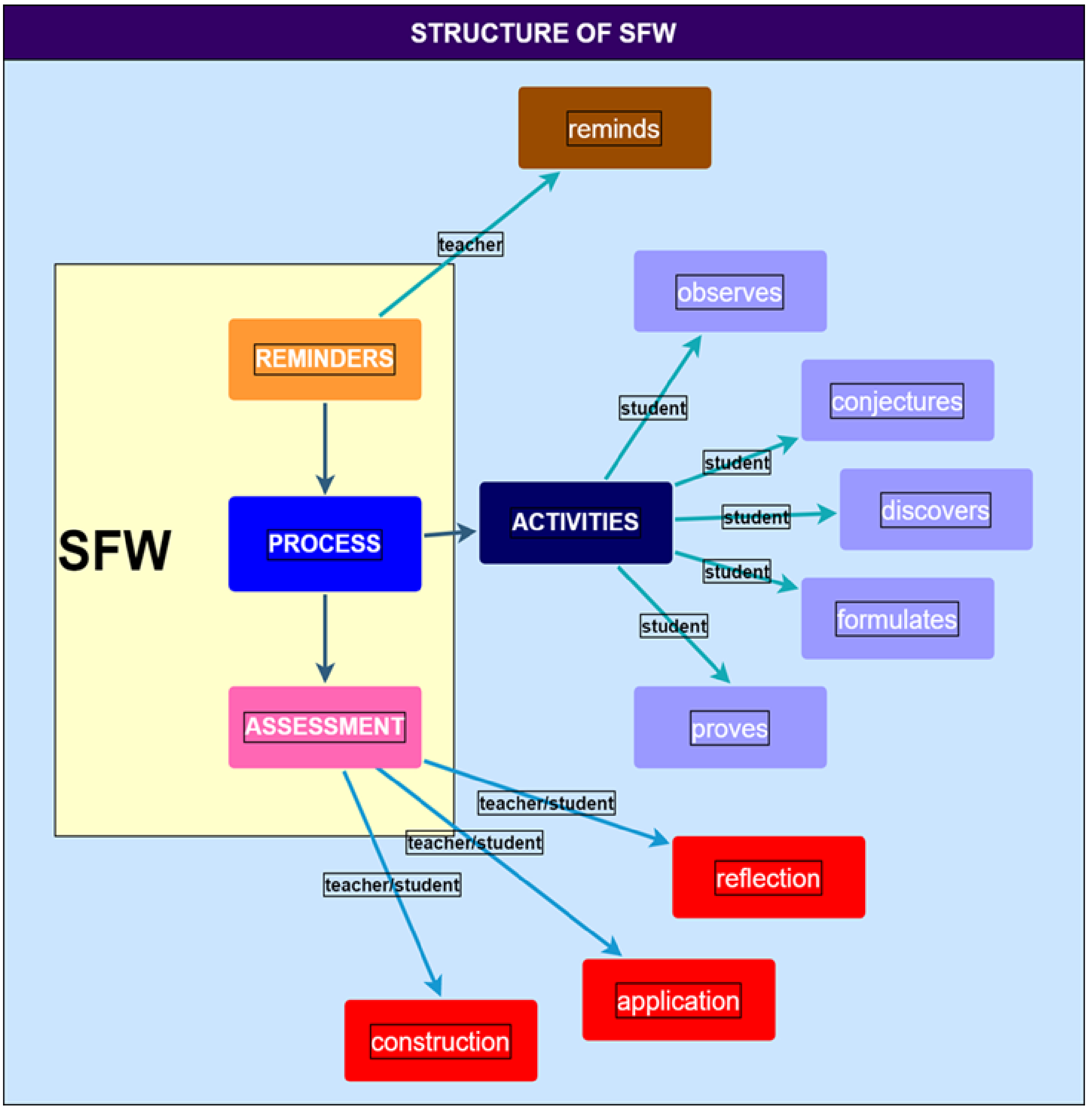
Worlwide, there have been numerous research studies that examine the difficulties encountered in the learning of geometry (Pyshkalo, 1968; Usiskin, 1982; Leron, 1985; Senk, 1985; Schoenfeld, 1985; Hoffer, 1986; Herscovics, 1989a; Mason, 1997; Harel & Sowder, 1998; Hanna, 2000; Weber, 2006; Dimakos & Nikoloudakis, 2008). In this paper, on the one hand we focus on van Hiele's theory of geometric thinking, as it has been described in his book "Structure and Insight" (Van Hiele, 1986), and on the other hand we focus on the Cognitive Apprenticeship theory (Collins, Brown & Newman, 1989), because we rely on these theoretical constructs to develop a new instructional model, entitled 'the Model of p-m Combinations". Moreover, stereometry, as a field of study, represents an ideal incentive for the students of primary education to obtain rich spatial experience and to develop effective spatial skills. In other words, it seems to be an appropriate method for providing an empirical understanding of the space and some of its forms to the students of primary education. Particularly, the study of stereometry may help these students to recognize the three-dimensional shapes, to learn how to draw them, to perceive and understand their nets etc. Nonetheless, the teaching and learning of stereometry is considered to be difficult by students, teachers and researchers due to the existence of three-dimensional figures. To this end, we employed ICT to overcome such kind of difficulties.

2. THE MODEL OF P-M COMBINATIONS

The Model of p-m Combinations (Nikoloudakis, 2009), represents an instructional model referring to the teaching of geometry. This instructional model came up after we combined the phases of the van Hiele theory with the methods of Cognitive Apprenticeship and enriched these combinations by adding the three following elements: (a) a special worksheet, entitled 'Structured Form Worksheet' (SFW), that we employed to teach geometry, (b) a matrix, entitled 'Reasoning Control Matrix for the Proving Process' (RE.CO.M.P.P.), that helped students in reasoning production, and (c) the concepts of simple and partial proof to write a formal proof. The 'Model of p-m Combinations' has been used in several research efforts, under different settings and contexts spanning from primary education to tertiary education. In Nikoloudakis (2009) there exists a more analytical description and explanation of the 'Model of p-m Combinations', through an empirical research to first-year senior high-school (Lyceum) students in Greece.

3. THE STRUCTURED FORM WORKSHEET

It is necessary here to describe SFW, as it is mentioned in Nikoloudakis (2009), because we will use it later on in this paper to improve the understanding of stereometry among 12-year old students. Additionally, the SFW represents an integral part of the aforementioned 'Model of p-m Combinations' to teach geometry to first-year senior high school students. 'Structured Form Worksheet' (Dimakos, Nikoloudakis, 2008) is a special worksheet, that consists of the following three sections: 1. The 'Reminders' Section 2. The 'Process' Section, and 3. The 'Assessment' Section (see Figure below).



Here, in this paper, we focus on students of primary education, therefore the SFW employed will definitely contain ICT-based learning activities aimed at 12-year old students of primary education. We certainly believe that the features of gamification, visualization, and dynamic manipulation of geometric shapes, that modern ICT can afford, significantly boost primary education students' understanding of the spatial geometry concepts. We must also note that the SFW contains the RECOMPP. RECOMPP (Dimakos et.al, 2007; Nikoloudakis, 2009) is a valuable tool that helps novice students to get familiarized with the process of formal proofs. We intentionally omitted to employ RECOMPP in our didactical scenario with the students of primary education, because 'Proofs' are not part of the Official Curriculum of Primary Education in Greece.

A more analytical description of SFW Sections follows:

1. The Reminder Notes*:* here, the teacher reminds the students of some theorems. These are some essential theorems, based on students' prior knowledge, that help them to understand the newly acquired knowledge. In this section, a combination, of the first phase of van Hiele's model (Inquiry/Information) with the method of Modeling of the Cognitive Apprenticeship model takes place.

2. The Process: here, the students have to conjecture, to discover, to argue, to prove, and to express their opinion on how to solve certain problems, that the teacher has prepared for them beforehand.

3. The Assessment: here, the students have to tell each other what they have done in the prior section, they have to describe the way they have thought, why they have thought this way, what they have learned etc. Also, they have to describe over the phone what they have learned to another schoolmate, who was absent from class. Moreover, students have to construct a problem based on the knowledge that they have gained. This section results from the combination of the fifth phase of van Hiele's model (Integration) with the method of Reflection of the Cognitive Apprenticeship model.

4. THE ROLE OF ICT IN THE UNDERSTANDING OF STEREOMETRY AMONG PRIMARY EDUCATION STUDENTS

As we have already mentioned earlier in the introduction of this paper, the study of stereometry helps primary students to develop their perceptual skills and also to practice their imagination ability regarding three-dimensional space. Hence, the selection and subsequent use of appropriate open problems e.g "Find what shape we get if we remove a part of a cone", seems to be an important prerequisite for the successful understanding of stereometry in primary education. It is worth mentioning here, that it is of extremely interest that case of the aforementioned open problem, when it is not clear which one of the parts of the cone is removed. Indeed, this specification transforms the problem into an open one, so it helps students to develop their critical thinking (Koleza, 2000; NCTM, 2000). (Abramovich, 2014; Freiman & Tassell, 2018) investigated the potential of technology to foster creative and divergent mathematical thinking, problem solving and problem posing, creative use of dynamic, multimodal and interactive software by teachers and learners, as well as other digital media and tools while widening and enriching transdisciplinary and interdisciplinary connections in mathematics classroom (Laborde, 2000). Carreira et. al (2016) mention that the different ways students deal with the tool and with mathematical knowledge are interpreted as instances of students-with-media, engaged in a "solving-with-dynamic-geometry-software" activity, enclosing a range of procedures brought forth by the symbioses between the affordances of the dynamic geometry software and the youngsters' aptitudes.

Obviously students' ability to perceive the net of a solid requires that they are aware of the plane shapes. However, students in primary education, find it quite more difficult, at least in terms of the shapes' perspective, to draw three-dimensional objects and in particular solid shapes in two-dimensional space e.g. on a chalkboard or on a piece of paper, than to draw plane shapes in a two-dimensional space. Indeed, the computers, via the use of ICT, may help significantly for this purpose. Also, modern ICT may help students of primary education to develop verbal skills, and communication skills, too. Also, Nikoloudakis et. al. (2019) corroborate that the computer environment contributes, by the alternation of the language used -since the educational software uses its own language of communication and interaction- to the transformation of the intra-course communication, while simultaneously leading to improved students' performance in the learning teaching process, compared to the performance of students when participating in a traditional, face-to-face, behavioral teaching.

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ICT have become one of the fundamental building blocks of modern society. Many researchers have carried out studies to evaluate the benefits of using ICT in mathematics. Based on selected literature, it was found that the advantages of applying ICT in teaching mathematics can be summarized in the following (Palmer, 2003; Keong, Sharaf & Daniel, 2005; Condie & Munro, 2007):

- it attracts students' interest in learning mathematics
- it increases their motivation and performance
- it encourages lifelong learning, and
- it facilitates positive interactions and relationships.

Ittigson & Zewe (2003) cited that technology is also essential in teaching and learning mathematics. ICT improves the way mathematics should be taught and enhances student understanding of basic concepts. Becta (2003) summarised the key benefits – ICT promotes greater collaboration among students and encourages communication and the sharing of knowledge. Many countries now regard the mastering of the basic skills and concepts of ICT as an inevitable part of the core education (Zakaria & Khalid, 2016). To this end, various new models of education are evolving in response to the new opportunities that are becoming available by integrating ICT and in particular Web-based technologies, into the teaching and learning environment (Ghavifekr et.al, 2012).

The effective integration of such applications in mathematics however, depends to a large extent on teacher's familiarity and ability with the IT learning environment (Ittigson & Zewe, 2003). Mathematics teachers need to know exactly how ICT is used as a teaching and learning tool, for their own purposes and to help students to use them. To get the most out of this process, mathematics teachers should be able to use ICT efficiently in research, problem solving and project-based learning in Mathematics, use ICT efficiently for professional development in the context of teaching and learning Mathematics, and integrate ICT appropriately into Mathematics curriculum activities that will foster students ownership of their ICT-rich learning environment. Representations and especially dynamic geometry software (DGS), i.e. those introduced by software such as Cabri, represent a special type of images that can be dragged and transformed under the effect of dragging.

In other words, Mariotti (2003) claims, that a tool of semiotic mediation is created, according to Vygotsky's perspective. Balacheff & Gaudin (2010) also claim that the transition from the computer's screen to real mathematics represents a process of modelling, too. This transition can also be done using learning objects from the Internet (Wiley, 2000; Nikoloudakis & Dimakos, 2009). Particularly, students can occupy themselves with those nets of solids that have a small number of faces e.g. the net of a cube or that of a tetrahedron, in order to develop the appropriate mental form through an empirical approach. Specifically, the students may draw, fold, or unfold these nets so as to develop the appropriate mental shape through an empirical approach.

The systematic use of special geometric software contributes to the transition from the passive learning to a more active one. In this way, the technology basically activates the learning context through the use of interactive software, by offering new roles to teachers and students (Chapman & Mahlck, 2004; Smeets, 2005). Particularly, the teacher undertakes to provide learning opportunities and subsequent learning activities by guiding and providing feedback to the students, while enabling students to express themselves (Kaware & Sain, 2015). Therefore, the teacher strengthens students' autonomy and encourages them in their learning efforts (Tailor & Tailor, 2007).

Moreover, nowadays innovative paradigms and models of collaborative learning design, suitable for electronic and distance learning, have been proposed (Paraskeva, Mysirlaki & Choustoulakis, 2009). Due to the convenience and flexibility of these new learning environments, the resources are available from anywhere and at any time. Web-based learning promotes active and independent learning, since everyone learns at their own pace (Palmer, 2003; Komis, 2004). In a classroom, where all students are being together, following the lesson may be difficult. This is a serious disadvantage of traditional education. On the other hand, online learning environments tend to solve this issue (Khalid, 2014). Indeed, in online education, all course materials are provided beforehand, and students learn by taking their own time (Kelley, 2011). They can also clarify their doubts by live chats or forums as well.

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Moreover, the ICT-based learning environments tend to favor students with different styles of learning, because ICT can provide diverse options for taking in and processing information, making sense of ideas, and expressing learning (Cheung & Slavin, 2013). The vast majority of students learn best through visual and tactile modalities, and ICT can help these students 'experience' the information instead of just reading and hearing it. Mobile devices can also offer programmes, aka "applications", that provide extra support to students with special needs, with features such as simplified screens and instructions, consistent placement of menus and control features, graphics combined with text, audio feedback, ability to set pace and level of difficulty, appropriate and unambiguous feedback, and easy error correction (Newton & Dell, 2011).

5. THE ICT-BASED DIDACTICAL SCENARIO BASED ON SFW

Structured Form Worksheet

Students' Team Names	**Teacher's Name**
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…………………………………………….	
…………………………………………….	
Class ……………………………………………	
School ……………………………………………	
Date ………………………………………	

Lesson Title : Nets of Solid Shapes

A. Reminders

The introduction is implemented using a playful approach. The students are asked to practice dealing with learning activities about plane shapes. These shapes represent faces of the nets of those solids to be taught. In particular, the students must do the following: (a) to recognize what kind of shape every net represents, and (b) to name the recognized shapes.

1. Visit the following webpage:

http://www.primarygames.com/puzzles/match_up/shape_match/shape_match.htm#

Play and try to find the similar shapes (see Fig. 1 below).

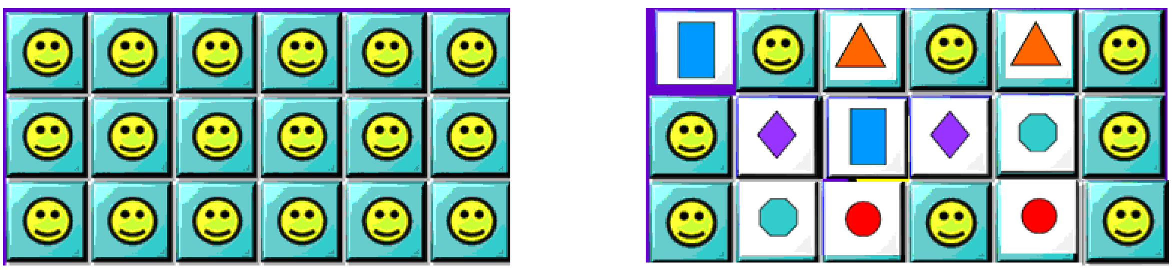

Fig.1 Screenshots from the learning activity: here, students play, by clicking on the smiley faces, attempting to find the similar shapes

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2. After you discover all the similar shapes, try to make a name list of these shapes

List below the names of the shapes
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…………………………………………
…………………………………………
…………………………………………
…………………………………………

3. Visit the following webpage:

http://www.mathsforyou.gr/index.php?option=com_wrapper&view=wrapper&Itemid=88

Here you can find another one game. Follow the available instructions (see Figure 2 below).

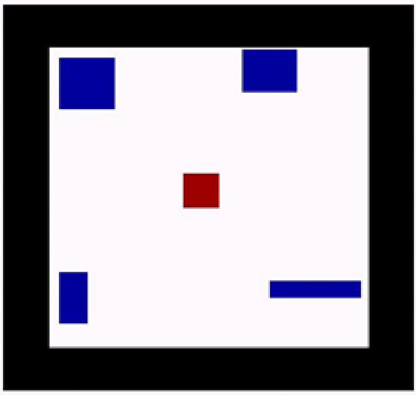

Fig. 2 Screenshot from the learning activity

4. Visit the following webpage:

http://arcytech.org/java/patterns/patterns_j.shtml

Draw your own objects, using the available shapes on the left panel. Then, you can name the objects you drawn.

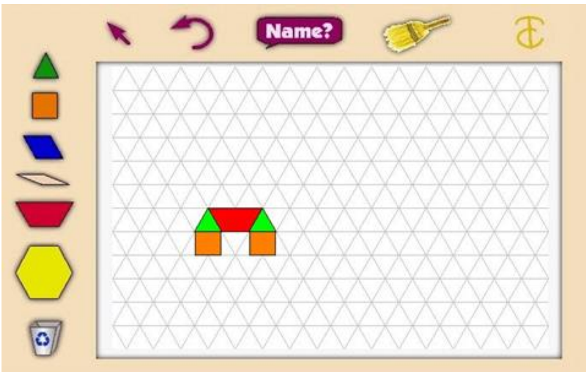

Fig. 3

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B. Process

1. Activity #1: Visit the following webpage:

http://www.edumedia-sciences.com/en/a570-nets-of-a-cube

Look carefully at the following nets of a cube.

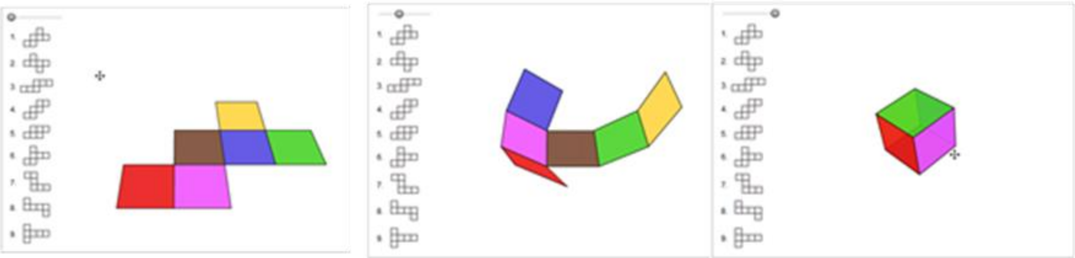

Fig. 4

Which are the plane shapes the net of a cube consists of?

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……………………………………………………………………………………………...
……………………………………………………………………………………………...

How many are the plane shapes the net of a cube consists of and which are they?

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……………………………………………………………………………………………...
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2. Activity #2: Draw, cut and fold the following net.

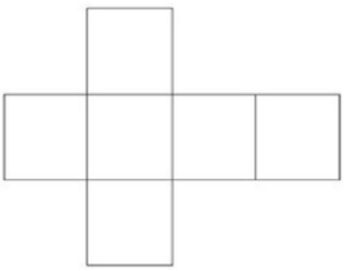

Fig. 5

3. Activity #3: Visit the following webpage:

http://www.edumedia-sciences.com/en/a569-net-of-a-polyhedron

Look carefully at the following nets of the rectangular parallelepiped.

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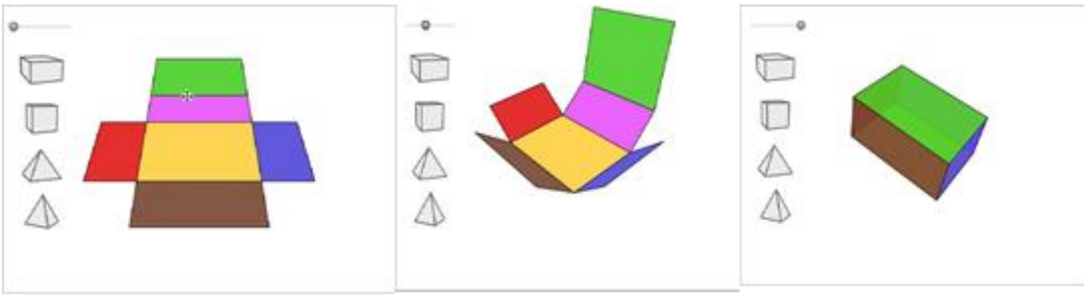

Fig. 6

How many are the plane shapes the net of a rectangular parallelepiped consists of and which are they?

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4. Activity #4: Draw, cut, and fold the following net.

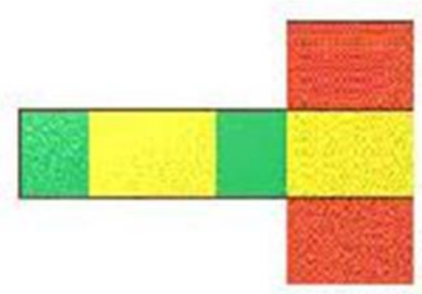

Fig. 7

5. Activity #5: Visit the following webpage:

http://www.edumedia-sciences.com/en/a569-net-of-a-polyhedron

Look carefully at the following nets of the rectangular puramid.

Fig. 8

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How many are the plane shapes the net of a rectangular based pyramid consists of and which are they?

………………………………………………………………………………………………...

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………………………………………………………………………………………………...

Can you imagine how many are the plane shapes the net of a hexagonal based pyramid consists of and which are they? ………………………………………………………………………………………………...

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6. Activity #6: Draw, cut, and fold the following net.

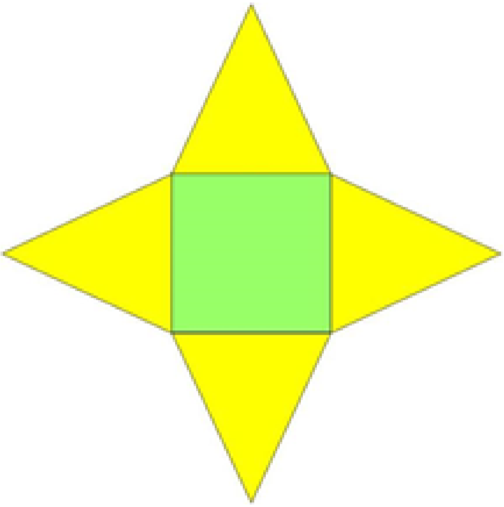

Fig. 9

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7. Activity #7: Visit the following webpage:

http://skoool.gr/content/los/primary/maths/3D_shapes/launch.html

In that webpage you will learn what a face, an edge, or an acne of a solid is about.

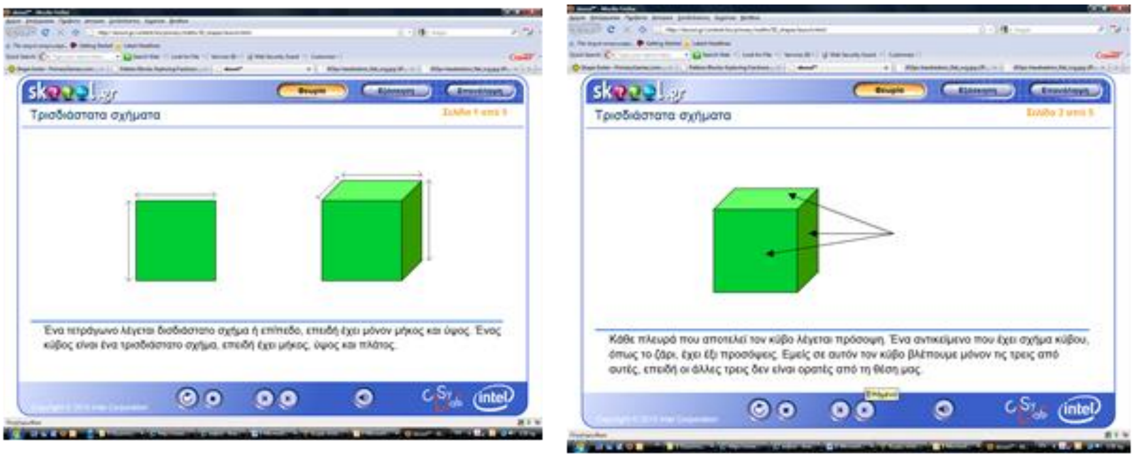

Fig. 10

8. Activity #8: Visit www.google.com and search for four cuboids. Then, count the edges, the acnes, and the faces of these cuboids, and then list them down in the following table.

Cuboid	# of faces	# of edges	# of acnes

C. Assessment

Activity #1: How many different colours you need to paint all the faces of the cube you drawed earlier, based on the given net? Do the same also for the pyramid.

Activity #2: Search in www.google.com for objects that correspond to every solid you learned. Write the name of each solid below its figure.

Activity #3: Send an e-mail to your classmates, describing them what you learned today.

Activity #4: Try to explain to a friend of yours, who has not understood the lesson, what the solids are.

6. CONCLUDING REMARKS AND FUTURE RESEARCH

The use of ICT in teaching stereometry can make the teaching process more effective as well as enhance the students' capabilities in understanding basic concepts. Nevertheless, implementing its use in teaching is not without problems as numerous barriers may arise (Jones, 2004). The types of barriers that have been identified in the study are the following: 1) lack of time in the school schedule for projects involving ICT 2) insufficient teacher training opportunities for ICT projects 3) inadequate technical support for these projects 4) lack of knowledge about ways to integrate ICT to enhance the Curriculum 5) difficulty in integrating and using different ICT tools in a single lesson, and 6) unavailability of resources at home for the students to access the necessary educational materials. A proposed method to overcome some of these barriers will be presented in a future research as part of this on-going research project on the improvement of geometry understanding for students of primary education.

Also, one issue that can be highlighted in future research is the effectiveness of the implementation of various ICT applications for improved student understanding. Besides, educators' ICT skills and teaching pedagogy can also be promoted as helping teachers believe that ICT can facilitate teaching as well as boost morale for students. The interest and motivation of students toward learning will increase if the teaching methods are not boring and provide interesting inputs each time the learning process occurs. Further studies on students' perceptions of computer-aided learning are also encouraged in order for educators to identify computer software or technology that suits the teaching of mathematics in modern schools.

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